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# On the biased Two-Parameter Estimator to Combat Multicollinearity in Linear Regression Model

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## Abstract

The most popularly used estimator to estimate the regression parameters in the linear regression model is the ordinary least-squares (OLS). The existence of multicollinearity in the model renders OLS inefficient. To overcome the multicollinearity problem, a new two-parameter estimator, a biased two-parameter (BTP), is proposed as an alternative to the OLS. Theoretical comparisons and simulation studies were carried out. The theoretical comparison and simulation studies show that the proposed estimator dominated some existing estimators using the mean square error (MSE) criterion. Furthermore, the real-life data bolster both the hypothetical and simulation results. The proposed estimator is preferred to OLS and other existing estimators when multicollinearity is present in the model.

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## 1. Introduction

Consider a linear regression model

$$y = X\beta + \varepsilon \tag{1}$$

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where $y$ is a $n \times 1$ vector of the dependent variable, X is a known n × p full rank matrix of explanatory variables, β is a p × 1 vector of regression coefficients and $\varepsilon \sim N\left(0, \sigma^2\right)$. The ordinary least squares estimator (OLS) of β in model 1 is defined as:

$$\hat{\beta}_{OLS} = (X'X)^{-1} X'y \tag{2}$$

The Ordinary Least Squares (OLS) estimator is the best linear unbiased estimator possessing minimum variance if the usual assumptions of a classical linear regression model are satisfied. Gujarati [1], Lukman [2], Ayinde et al. [3]. However, when the column vectors of the regressors are linearly dependent, known as multicollinearity, the OLSE becomes unstable. OLS produces unbiased but inefficient estimates of the model parameters [3].

Biased estimators such as Stein estimator [4], Principal Component estimator [5], Partial Least Square estimator [6], Ridge regression estimator [7], Liu estimator [8], modified ridge regression estimator [9], two-parameter (TP) estimator [10], KL estimator [11], DK estimator [12], proposed two-parameter estimator [13], Liu-Dawoud-Kibria estimator [14], New ridge-type estimator [15], Generalized Kibria-Lukman estimator [16], Tobit new ridge-type [17] and others are often used to address this problem.

The objective of this paper is to introduce a new two-parameter estimator to address the problem of multicollinearity in linear regression and to compare the performance of the new estimator with the OLS estimator, the ordinary ridge regression (ORR) estimator [7], the Liu estimator [8], the modified one-parameter Liu (ML) estimator [18] and the Dawoud-Kibria (DK) estimator [12]. Then, below is a short description of each estimator to compare with the new estimator.

The canonical form of (1) can be written as:

$$y = Z\alpha + u \tag{3}$$

where Z=XQ, $\alpha = Q'\beta$ and Q is the orthogonal matrix such that $Z'Z = Q'X'XQ = \Lambda = diag\left(\lambda_1, \lambda_2, ..., \lambda_p\right)$. The OLS estimator of $\alpha$is given as follows:

$$\hat{\alpha} = \Lambda^{-1} Z'y \tag{4}$$

And the mean squared error matrix (MSEM) of $\hat{\alpha}$is given by

$$MSEM(\hat{\alpha}) = \sigma^2 \Lambda^{-1} \tag{5}$$

The ORR estimator $\alpha$ by Hoerl and Kennard [7] is defined as:

$$\hat{\alpha}_k = MX'y \tag{6}$$

where $M = (\Lambda + kI)^{-1}$ and k is a biasing parameter such that k > 0.

$$MSEM(\hat{\alpha}_k) = \sigma^2 M\Lambda M' + (M\Lambda - I)\alpha\alpha'(M\Lambda - I)' \tag{7}$$

The Liu estimator by Liu [8] is defined as:

$$\hat{\alpha}_d = S\hat{\alpha} \tag{8}$$

where $S = (\Lambda + I)^{-1}(\Lambda + dI)$ and d is a biasing parameter of the Liu Estimator.

$$MSEM(\hat{\alpha}_d) = \sigma^2 S\Lambda^{-1} S' + (S - I)\alpha\alpha'(S - I)' \tag{9}$$

The Modified One-Parameter Liu estimator by Lukman *et al.* [18]. is defined as:

$$\hat{\alpha}_{ML} = E\hat{\alpha} \tag{10}$$

where $E = (\Lambda + I)^{-1}(\Lambda - dI)$ and d is a biasing parameter of the Liu Estimator.

$$MSEM(\hat{\alpha}_{ML}) = \sigma^2 E\Lambda^{-1} E' + (E - I)\alpha\alpha'(E - I)' \tag{11}$$

The Kibria-Lukman (KL) estimator [11] is defined as:

$$\hat{\alpha}_{KL} = F\hat{\alpha} \tag{12}$$

where $F = (\Lambda + kI)^{-1}(\Lambda - kI)$ and

$$MSEM(\hat{\alpha}_{KL}) = \sigma^2 F\Lambda^{-1}F' + (F - I)\alpha\alpha'(F - I)' \tag{13}$$

The Dawoud-Kibria (DK) estimator [12] is defined as:

$$\hat{\alpha}_{DK} = H\hat{\alpha} \tag{14}$$

where $H = (\Lambda + k(1 + d)I)^{-1}(\Lambda - k(1 + d)I)$ and

$$MSEM(\hat{\alpha}_{DK}) = \sigma^2 H\Lambda^{-1}H' + (H - I)\alpha\alpha'(H - I)' \tag{15}$$

**The proposed estimator**

The Modified One-Parameter Liu [18] estimator $\alpha$is defined as:

$$\hat{\alpha}_{ML} = E\hat{\alpha}$$

Following the similar method proposed by Liu [19] and Kaciranlar *et al.* [20], the proposed Biased Two-Parameter estimator $\alpha_{BTP}$ is defined as:

$$\hat{\alpha}_{BTP} = EH\hat{\alpha} \tag{16}$$

Properties of the new estimator

The bias, covariance, and mean squared error matrix (MSEM) of the proposed estimator are given as follows:

$$B(\hat{\alpha}_{BTP}) = (EH - I)\alpha \tag{17}$$

$$D(\hat{\alpha}_{BTP}) = \sigma^2 EH\Lambda^{-1}E'H' \tag{18}$$

$$MSEM(\hat{\alpha}_{BTP}) = \sigma^2 EH\Lambda^{-1}E'H' + (EH - I)\alpha\alpha'(EH - I)' \tag{19}$$

The following lemmas are useful for making some theoretical comparisons and proving the statistical properties $\hat{\alpha}_{BTP}$.

**Lemma 1**: Let J be an n x n positive definite matrix, that is, J > 0, and $\alpha$be some vector, then $J - \alpha\alpha' \geq 0$ if and only if $\alpha' J^{-1}\alpha \leq 1$. Farebrother [21]

**Lemma 2**: Let $\hat{\alpha}_i = A_i y$ i = 1, 2 be two linear estimators of $\alpha$. Suppose that $D = cov(\hat{\alpha}_1) - cov(\alpha_2) > 0$, where $cov(\hat{\alpha}_i)$ i = 1, 2 denotes the covariance matrix of $\hat{\alpha}_i$ and $b_i = Bias(\hat{\alpha}_i) = (A_i X - I)\alpha$, i =1, 2. Consequently,

$$\Delta(\hat{\alpha}_1 - \hat{\alpha}_2) = MSEM(\hat{\alpha}_1) - MSEM(\hat{\alpha}_2) = \sigma^2 D + b_1 b_1^{'} - b_2 b_2^{'} > 0 \tag{20}$$

If and only if $b_2^{'}\left[\sigma^2 D + b_1 b_1^{'}\right]^{-1} b_2 < 1$, where MSEM$(\hat{\alpha}_i) = cov(\hat{\alpha}_i) + b_i b_i^{'}$ Trenkler and Toutenburg [22]

The article was organized as follows. In section 2, a theoretical comparison of the proposed estimator with the existing ones is done. In section 3, biasing parameters are derived. A simulation study is conducted in Section 4 to evaluate the performance of the proposed estimator. A numerical example is given in Section 5 to illustrate the findings in the paper, and Section 6 ends with some concluding remarks.

## 2. Comparison among the Estimators

In this section, theoretical comparisons among the proposed BTP estimator and the OLS, ORR, Liu, KL, ML, and DK estimators

*Comparison between $\hat{\alpha}$ and $\hat{\alpha}_{BTP}$.*

The difference between $MSEM(\hat{\alpha}) - MSEM(\hat{\alpha}_{BTP})$ is given as follows:

$$MSEM(\hat{\alpha}) - MSEM(\hat{\alpha}_{MLKL}) = \sigma^2\Lambda^{-1} - \left(\sigma^2 EH\Lambda^{-1}E'H' + (EH - I)\alpha\alpha'(EH - I)'\right) \tag{21}$$

Let $k > 0$ and $0 < d < 1$. Thus, the following theorem holds.

**Theorem 1:** The estimator $\hat{\alpha}_{BTP}$is superior to the estimator $\hat{\alpha}$ if and only if

$$\alpha'(EH-I)'\left\lfloor\sigma^2\left(\Lambda^{-1}-EH\Lambda^{-1}E'H'\right)\right\rfloor\alpha(EH-I)<1 \tag{22}$$

Proof

$$\begin{aligned} D[\hat{\alpha}]-D(\hat{\alpha}_{BTP}) &= \sigma^2\left(\Lambda^{-1}-EH\Lambda^{-1}E'H'\right) \\ &= \sigma^2 diag\left\{\frac{1}{\lambda_i}-\frac{(\lambda_i-d)^2(\lambda_i-k(I+d))^2}{\lambda_i(\lambda_i+1)^2(\lambda_i+k(1+d))^2}\right\}_{i=1}^{p} \end{aligned} \tag{23}$$

$\Lambda^{-1}-EH\Lambda^{-1}E'H'$ will be pdf if and only if $(\lambda_i+1)^2(\lambda_i+k(1+d)^2-(\lambda_i-d)^2(\lambda_i-k(1+d))^2>0$.

*Comparison between $\hat{\alpha}_k$ and $\hat{\alpha}_{BTP}$.*

The difference between $MSEM(\hat{\alpha}_k)-MSEM(\hat{\alpha}_{BTP})$ is given as follows:

$$\begin{aligned} &MSEM(\hat{\alpha}_k)-MSEM(\hat{\alpha}_{BTP}) \\ &= \left(\sigma^2M\Lambda M'+(M\Lambda-I)\alpha\alpha'(M\Lambda-I)'\right)-\left(\sigma^2EH\Lambda^{-1}E'H'+(EH-I)\alpha\alpha'(EH-I)'\right) \end{aligned} \tag{24}$$

Let $k>0$ and $0<d<1$. Thus, the following theorem holds.

**Theorem 3.2:** The estimator $\hat{\alpha}_{BTP}$is superior to the estimator $\hat{\alpha}_k$using the MSEM criterion, that is, MSEM $(\hat{\alpha}_k)$ − MSEM $(\hat{\alpha}_{BTP})>0$ if and only if

$$\alpha'(EH-I)'\left\lfloor\sigma^2\left(M\Lambda M'-EH\Lambda^{-1}E'H'\right)+(M\Lambda-I)\alpha\alpha'(M\Lambda-I)'\right\rfloor\alpha(EH-I)<1 \tag{25}$$

Proof

$$\begin{aligned} D(\hat{\alpha}_k)-D(\hat{\alpha}_{BTP}) &= \sigma^2\left(M\Lambda M'-EH\Lambda^{-1}E'H'\right) \\ &= \sigma^2 diag\left\{\frac{\lambda_i}{(\lambda_i+k)^2}-\frac{(\lambda_i-d)^2(\lambda_i-k(1+d))^2}{\lambda_i(\lambda_i+1)^2(\lambda_i+k(1+d))^2}\right\}_{i=1}^{p} \end{aligned} \tag{26}$$

$M\Lambda M-EH\Lambda^{-1}E'H'$ will be pdf if and only if $\lambda_i^2(\lambda_i+1)^2(\lambda_i+k(1+d))^2-(\lambda_i-d)^2(\lambda_i-k)^2(\lambda_i+k)^2>0$.

*Comparison between $\hat{\alpha}_d$ and $\hat{\alpha}_{BTP}$.*

The difference between $MSEM(\hat{\alpha}_d)-MSEM(\hat{\alpha}_{BTP})$ is given as follows:

$$\begin{aligned} &MSEM(\hat{\alpha}_d)-MSEM(\hat{\alpha}_{BTP}) \\ &= \left(\sigma^2S\Lambda^{-1}S'+(S-I)\alpha\alpha'(S-I)'\right)-\left(\sigma^2EH\Lambda^{-1}E'H'+(EH-I)\alpha\alpha'(EH-I)'\right) \end{aligned} \tag{27}$$

Let $k>0$ and $0<d<1$. Thus, the following theorem holds.

**Theorem 3.3:** The estimator $\hat{\alpha}_{BTP}$is superior to the estimator $\hat{\alpha}_d$using the MSEM criterion, that is, $MSEM(\hat{\alpha}_d)-MSEM(\hat{\alpha}_{BTP})>0$if and only if

$$\alpha'(EH-I)'\left\lfloor\sigma^2\left(S\Lambda^{-1}S'-EH\Lambda^{-1}E'H'\right)+(S-I)\alpha\alpha'(S-I)'\right\rfloor\alpha(EH-I)<1 \tag{28}$$

Proof

$$\begin{aligned} D[\hat{\alpha}_d]-D(\hat{\alpha}_{BTP}) &= \sigma^2\left(S\Lambda^{-1}S'-EH\Lambda^{-1}E'H'\right) \\ &= \sigma^2 diag\left\{\frac{(\lambda_i+d)^2}{\lambda_i(\lambda_i+1)^2}-\frac{(\lambda_i-d)^2(\lambda_i-k(1+d))^2}{\lambda_i(\lambda_i+1)^2(\lambda_i+k(1+d))^2}\right\}_{i=1}^{p} \end{aligned} \tag{29}$$

$S\Lambda^{-1}S'-EH\Lambda^{-1}E'H'$ will be pdf if and only if $(\lambda_i+d)^2(\lambda_i+k(1+d))^2-(\lambda_i-d)^2(\lambda_i-k(1+d))^2>0$.

*Comparison between $\hat{\alpha}_{ML}$ and $\hat{\alpha}_{BTP}$.*

The difference between $MSEM(\hat{\alpha}_{ML})-MSEM(\hat{\alpha}_{BTP})$ is given as follows:

$$MSEM(\hat{\alpha}_{ML})-MSEM(\hat{\alpha}_{BTP})=\left(\sigma^2E\Lambda^{-1}E'+(E-I)\alpha\alpha'(E-I)'\right)-\left(\sigma^2EH\Lambda^{-1}E'H'+(EH-I)\alpha\alpha'(EH-I)'\right) \tag{30}$$

Let $k>0$ and $0<d<1$. Thus, the following theorem holds.

**Theorem 3.4:** The estimator $\hat{\alpha}_{BTP}$is superior to the estimator $\hat{\alpha}_{ML}$using the MSEM criterion, that is, MSEM $(\hat{\alpha}_{ML})$ − MSEM $(\hat{\alpha}_{BTP}) > 0$ if and only if

$$\alpha'(EH-I)'\left\lfloor\sigma^2\left(E\Lambda^{-1}E'-EH\Lambda^{-1}E'H'\right)+(E-I)\alpha\alpha'(E-I)'\right\rfloor\alpha(EH-I)<1 \tag{31}$$

Proof

$$D[\hat{\alpha}_{ML}]-D(\hat{\alpha}_{BTP})=\sigma^2\left(E\Lambda^{-1}E'-EH\Lambda^{-1}E'H'\right)$$

$$=\sigma^2 diag\left\{\frac{(\lambda_i-d)^2}{\lambda_i(\lambda_i+1)^2}-\frac{(\lambda_i-d)^2(\lambda_i-k(1+d))^2}{\lambda_i(\lambda_i+1)^2(\lambda_i+k(1+d))^2}\right\}_{i=1}^{p} \tag{32}$$

$E\Lambda^{-1}E'-EH\Lambda^{-1}E'H'$ will be pdf if and only if $(\lambda_i-d)^2(\lambda_i+k(1+d))^2-(\lambda_i-d)^2(\lambda_i-k(1+d))^2>0$.

*Comparison between $\hat{\alpha}_{KL}$ and $\hat{\alpha}_{BTP}$.*

The difference between $MSEM(\hat{\alpha}_{KL})-MSEM(\hat{\alpha}_{BTP})$ is given as follows:

$$MSEM(\hat{\alpha}_{KL})-MSEM(\hat{\alpha}_{BTP})=\left(\sigma^2F\Lambda^{-1}F'+(F-I)\alpha\alpha'(F-I)'\right)-\left(\sigma^2EH\Lambda^{-1}E'H'+(EH-I)\alpha\alpha'(EH-I)'\right) \tag{33}$$

Let $k>0$ and $0<d<1$. Thus, the following theorem holds.

**Theorem 3.5:** The estimator $\hat{\alpha}_{BTP}$is superior to the estimator $\hat{\alpha}_{KL}$using the MSEM criterion, that is, MSEM $(\hat{\alpha}_{KL})$ − MSEM $(\hat{\alpha}_{BTP}) > 0$ if and only if

$$\alpha'(EH-I)'\left\lfloor\sigma^2\left(F\Lambda^{-1}F'-EH\Lambda^{-1}E'H'\right)+(F-I)\alpha\alpha'(F-I)'\right\rfloor\alpha(EH-I)<1 \tag{34}$$

Proof

$$D[\hat{\alpha}_{KL}]-D(\hat{\alpha}_{BTP})=\sigma^2\left(F\Lambda^{-1}F'-EH\Lambda^{-1}E'H'\right)$$

$$=\sigma^2 diag\left\{\frac{(\lambda_i-k)^2}{\lambda_i(\lambda_i+k)^2}-\frac{(\lambda_i-d)^2(\lambda_i-k(1+d))^2}{\lambda_i(\lambda_i+1)^2(\lambda_i+k(1+d))^2}\right\}_{i=1}^{p} \tag{35}$$

$F\Lambda^{-1}F'-EH\Lambda^{-1}E'H'$ will be pdf if and only if $(\lambda_i-k)^2(\lambda_i+1)^2(\lambda_i+k(1+d))^2-(\lambda_i-d)^2(\lambda_i-k(1+d))^2(\lambda_i+k)^2>0$.

*Comparison between $\hat{\alpha}_{DK}$ and $\hat{\alpha}_{BTP}$.*

The difference between $MSEM(\hat{\alpha}_{DK})-MSEM(\hat{\alpha}_{BTP})$ is given as follows:

$$MSEM(\hat{\alpha}_{DK})-MSEM(\hat{\alpha}_{BTP})=\left(\sigma^2H\Lambda^{-1}H'+(H-I)\alpha\alpha'(H-I)'\right)-\left(\sigma^2EH\Lambda^{-1}E'H'+(EH-I)\alpha\alpha'(EH-I)'\right) \tag{36}$$

Let $k>0$ and $0<d<1$. Thus, the following theorem holds.

**Theorem 3.6:** The estimator $\hat{\alpha}_{BTP}$is superior to the estimator $\hat{\alpha}_{DK}$using the MSEM criterion, that is, MSEM $(\hat{\alpha}_{DK})$ − MSEM $(\hat{\alpha}_{BTP}) > 0$ if and only if

$$\alpha'(EH-I)'\left\lfloor\sigma^2\left(H\Lambda^{-1}H'-EH\Lambda^{-1}E'H'\right)+(H-I)\alpha\alpha'(H-I)'\right\rfloor\alpha(EH-I)<1 \tag{37}$$

Proof

$$D[\hat{\alpha}_{DK}]-D(\hat{\alpha}_{BTP})=\sigma^2\left(H\Lambda^{-1}H'-EH\Lambda^{-1}E'H'\right)$$

$$=\sigma^2 diag\left\{\frac{(\lambda_i-k(1+d))^2}{\lambda_i(\lambda_i+k(1+d))^2}-\frac{(\lambda_i-d)^2(\lambda_i-k(1+d))^2}{\lambda_i(\lambda_i+1)^2(\lambda_i+k(1+d))^2}\right\}_{i=1}^{p} \tag{38}$$

$H\Lambda^{-1}H'-EH\Lambda^{-1}E'H'$ will be pdf if and only if $(\lambda_i-k(1+d))^2(\lambda_i+1)^2-(\lambda_i-d)^2(\lambda_i-k(1+d))^2>0$.

## 3. Selection of Biasing Parameters

For practical use, there is a need to estimate the parameters $k$ and $d$. Different researchers have proposed different estimators of k and d for different types of regression models. Following some of these researchers like Hoerl *et al*. [7], Liu [8], Lukman and Ayinde [23], Ayinde *et al.* [24], Owolabi *et al.* [25]. and Dawoud *et al*. [12, 26] to mention a few, among others. In determining the value of $k$, $d$ is fixed such that $k$ is obtained as follows:

$$MSEM\left(\hat{\alpha}_{BTP}\right) = \sigma^2 EH\Lambda^{-1}E'H' + \left(EH - I\right)\alpha\alpha'\left(EH - I\right)'$$

$$l(k,d) = MSEM\left(\hat{\alpha}_{BTP}\right) = tr\left[MSEM\left(\hat{\alpha}_{BTP}\right)\right]$$

$$l\left(k,d\right) = \sigma^2 \sum_{i}^{p} \frac{\left(\lambda_i - d\right)^2 \left(\lambda_i - k(1+d)\right)^2}{\lambda_i \left(\lambda_i + 1\right)^2 \left(\lambda_i + k(1+d)\right)^2} + \sum_{i}^{p} \frac{\left(2\lambda_i k + \lambda_i d - kd + \lambda_i + k(1+d)\right)^2 \alpha_i^2}{\left(\lambda_i + 1\right)^2 \left(\lambda_i + k(1+d)\right)^2} \tag{39}$$

Differentiating $l(k,d)$with respect to $k$ and setting $\frac{\partial l(k,d)}{\partial k} = 0$, we have

$$k = \frac{\sigma^2 \lambda_i \left(d - \lambda_i\right) + \alpha_i^2 \lambda_i^2 \left(d + 1\right)}{\sigma^2 \left(1 + d\right)\left(d - \lambda_i\right) - \alpha_i^2 \lambda_i \left(1 + d\right)\left(2\lambda_i - d + 1\right)} \tag{40}$$

Also, the value of $d$ is found by differentiating $l\left(k,d\right)$with respect to $d$; for a fixed $k$ and setting $\frac{\partial l(k,d)}{\partial d} = 0$, we obtain

$$\begin{aligned} d = {} & \frac{-\left(\sigma^2\lambda_i - \sigma^2 k + \alpha_i^2\lambda_i^2 + \sigma^2\lambda_i k + 2\alpha_i^2\lambda_i^2 k\right)}{2\left(\sigma^2 k + \alpha_i^2\lambda_i k\right)} \\ & + \frac{\sqrt{(\alpha_i^2)^2\lambda_i^2 k(2\lambda_i k + k + \lambda_i) + \sigma^2\alpha_i^2\lambda_i^2 k(k - \lambda_i) + \sigma^2\alpha_i^2\lambda_i k(2\sigma^2\lambda_i k + k + \lambda_i) + (\sigma^2)^2\lambda_i k(k - \lambda_i)}}{\left(\sigma^2 k + \alpha_i^2\lambda_i k\right)} \end{aligned} \tag{41}$$

For the practical purpose $\sigma$and $\alpha$are replaced with $\hat{\sigma}$and $\hat{\alpha}$respectively in (40) and (41)

$$\hat{k} = \frac{\hat{s}^2 \lambda_i \left(d - \lambda_i\right) + \hat{a}_i^2 \lambda_i^2 \left(d + 1\right)}{\hat{s}^2 \left(1 + d\right)\left(d - \lambda_i\right) - \hat{a}_i^2 \lambda_i \left(1 + d\right)\left(2\lambda_i - d + 1\right)} \tag{42}$$

$$\begin{aligned} \hat{d} = {} & \frac{-\left(\hat{\sigma}^2\lambda_i - \hat{\sigma}^2 k + \hat{\alpha}_i^2\lambda_i^2 + \hat{\sigma}^2\lambda_i k + 2\hat{\alpha}_i^2\lambda_i^2 k\right)}{2\left(\hat{\sigma}^2 k + \hat{\alpha}_i^2\lambda_i k\right)} \\ & + \frac{\sqrt{(\hat{\alpha}_i^2)^2\lambda_i^2 k(2\lambda_i k + k + \lambda_i) + \hat{\sigma}^2\hat{\alpha}_i^2\lambda_i^2 k(k - \lambda_i) + \hat{\sigma}^2\hat{\alpha}_i^2\lambda_i k(2\hat{\sigma}^2\lambda_i k + k + \lambda_i) + (\hat{\sigma}^2)^2\lambda_i k(k - \lambda_i)}}{\left(\hat{\sigma}^2 k + \hat{\alpha}_i^2\lambda_i k\right)} \end{aligned} \tag{43}$$

## 4. Simulation Techniques

To study the performance of the estimators, a simulation has been carried out in this section. Following McDonald and Galarneau [27], Gibbons [28], and Lukman *et al*. [29], the explanatory variables are generated using the equation:

$$x_{ij} = \left(1 - \rho^2\right)^{1/2} z_{ij} + \rho z_{i,p+1}, i = 1, 2, ..., n, \quad j = 1, 2, ..., p. \tag{44}$$

where $z_{ij}$ are independent standard normal pseudo-random numbers, ρ represents the correlation between two explanatory variables, and p is the number of explanatory variables. In this study, $\rho$ is considered to be 0.80, 0.90, 0.95, and 0.99. We consider p = 3 and 7 in the simulation. The following equation determines the $n$ observations for the dependent variable $y$:

$$Y_i = \beta_1 X_1 + \beta_2 X_2 + ... + \beta_p X_p + e_i \tag{45}$$

where $e_i$ is normally distributed with mean 0 and variance $\sigma^2$ Firinguetti [30]. According to Newhouse and Oman, we choose $\beta$such that $\beta'\beta$=1 [31]. The chosen values of $\sigma$are 3, 5, and 10, and the replication for the study is 1000 times. The mean square error is then obtained as:

$$MSE\left(\hat{\alpha}\right) = \frac{1}{1000} \sum_{j=1}^{1000} \left(\hat{\alpha}_{ij} - \alpha_i\right)' \left(\hat{\alpha}_{ij} - \alpha_i\right) \tag{46}$$

Table 1: Estimated MSE when n=50 and p=3

| K | d | sigma | rho | OLS | RIDGE | LIU | K-L | DK | ML | BTP |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.2 | 3 | 0.8 | 1.227 | 1.1802 | 1.1098 | 1.1344 | 1.1168 | 1.0535 | **0.9598** |
| | | | 0.9 | 2.2158 | 2.0587 | 1.8412 | 1.9076 | 1.8514 | 1.6674 | **1.3965** |
| | | | 0.95 | 4.2136 | 3.6579 | 2.9999 | 3.1428 | 2.964 | 2.4725 | **1.7516** |
| | | | 0.99 | 20.175 | 10.956 | 6.5009 | 4.5913 | 3.3199 | 2.5317 | **0.4895** |
| | | 5 | 0.8 | 3.4082 | 3.2782 | 3.0825 | 3.1509 | 3.1019 | 2.9261 | **2.6655** |
| | | | 0.9 | 6.1551 | 5.7185 | 5.1142 | 5.2987 | 5.1426 | 4.6314 | **3.8787** |
| | | | 0.95 | 11.704 | 10.161 | 8.333 | 8.73 | 8.2333 | 6.868 | **4.8656** |
| | | | 0.99 | 56.042 | 30.434 | 18.058 | 12.754 | 9.2219 | 7.0326 | **1.3598** |
| | | 10 | 0.8 | 13.633 | 13.113 | 12.329 | 12.603 | 12.407 | 11.704 | **10.661** |
| | | | 0.9 | 24.62 | 22.874 | 20.457 | 21.194 | 20.57 | 18.525 | **15.513** |
| | | | 0.95 | 46.818 | 40.644 | 33.332 | 34.92 | 32.933 | 27.472 | **19.463** |
| | | | 0.99 | 224.17 | 121.73 | 72.233 | 51.015 | 36.888 | 28.131 | **5.4392** |
| | 0.5 | 3 | 0.8 | 1.227 | 1.1802 | 1.153 | 1.1344 | 1.0909 | 1.0124 | **0.9015** |
| | | | 0.9 | 2.2158 | 2.0587 | 1.9774 | 1.9076 | 1.7703 | 1.543 | **1.2377** |
| | | | 0.95 | 4.2136 | 3.6579 | 3.4302 | 3.1428 | 2.7145 | 2.1117 | **1.3776** |
| | | | 0.99 | 20.175 | 10.956 | 10.733 | 4.5913 | 1.9692 | 0.8096 | **0.1418** |
| | | 5 | 0.8 | 3.4082 | 3.2782 | 3.2026 | 3.1509 | 3.03 | 2.8117 | **2.5032** |
| | | | 0.9 | 6.1551 | 5.7185 | 5.4928 | 5.2987 | 4.9173 | 4.2857 | **3.4373** |
| | | | 0.95 | 11.704 | 10.161 | 9.5284 | 8.73 | 7.5403 | 5.8658 | **3.8266** |
| | | | 0.99 | 56.042 | 30.434 | 29.813 | 12.754 | 5.4701 | 2.2488 | **0.3937** |
| | | 10 | 0.8 | 13.633 | 13.113 | 12.81 | 12.603 | 12.119 | 11.246 | **10.011** |
| | | | 0.9 | 24.62 | 22.874 | 21.971 | 21.194 | 19.669 | 17.142 | **13.748** |
| | | | 0.95 | 46.818 | 40.644 | 38.114 | 34.92 | 30.161 | 23.464 | **15.306** |
| | | | 0.99 | 224.17 | 121.73 | 119.25 | 51.015 | 21.881 | 8.9955 | **1.5749** |
| | 0.8 | 3 | 0.8 | 1.227 | 1.1802 | 1.1971 | 1.1344 | 1.0656 | 0.9721 | **0.8463** |
| | | | 0.9 | 2.2158 | 2.0587 | 2.1188 | 1.9076 | 1.6928 | 1.4237 | **1.0944** |
| | | | 0.95 | 4.2136 | 3.6579 | 3.8903 | 3.1428 | 2.4857 | 1.7807 | **1.0724** |
| | | | 0.99 | 20.175 | 10.956 | 16.04 | 4.5913 | 1.1027 | 0.1629 | **0.0567** |
| | | 5 | 0.8 | 3.4082 | 3.2782 | 3.3251 | 3.1509 | 2.9597 | 2.6997 | **2.3495** |
| | | | 0.9 | 6.1551 | 5.7185 | 5.8855 | 5.2987 | 4.702 | 3.9542 | **3.0389** |
| | | | 0.95 | 11.704 | 10.161 | 10.806 | 8.73 | 6.9046 | 4.9464 | **2.9786** |
| | | | 0.99 | 56.042 | 30.434 | 44.554 | 12.754 | 3.063 | 0.4524 | **0.1572** |
| | | 10 | 0.8 | 13.633 | 13.113 | 13.3 | 12.603 | 11.838 | 10.797 | **9.3953** |
| | | | 0.9 | 24.62 | 22.874 | 23.542 | 21.194 | 18.808 | 15.816 | **12.154** |
| | | | 0.95 | 46.818 | 40.644 | 43.226 | 34.92 | 27.619 | 19.786 | **11.914** |
| | | | 0.99 | 224.17 | 121.73 | 178.22 | 51.015 | 12.252 | 1.8099 | **0.6288** |
| 0.6 | 0.2 | 3 | 0.8 | 1.227 | 1.1362 | 1.1098 | 1.0491 | 1.0169 | 1.0535 | **0.8749** |
| | | | 0.9 | 2.2158 | 1.9183 | 1.8412 | 1.6431 | 1.548 | 1.6674 | **1.1708** |
| | | | 0.95 | 4.2136 | 3.208 | 2.9999 | 2.3436 | 2.0827 | 2.4725 | **1.2415** |
| | | | 0.99 | 20.175 | 6.9114 | 6.5009 | 0.721 | 0.2939 | 2.5317 | **0.0876** |
| | | 5 | 0.8 | 3.4082 | 3.1559 | 3.0825 | 2.9139 | 2.8244 | 2.9261 | **2.4293** |
| | | | 0.9 | 6.1551 | 5.3284 | 5.1142 | 4.5638 | 4.2996 | 4.6314 | **3.2513** |
| | | | 0.95 | 11.704 | 8.9111 | 8.333 | 6.5101 | 5.7852 | 6.868 | **3.4484** |
| | | | 0.99 | 56.042 | 19.198 | 18.058 | 2.0029 | 0.8163 | 7.0326 | **0.2433** |
| | | 10 | 0.8 | 13.633 | 12.623 | 12.329 | 11.655 | 11.296 | 11.704 | **9.7148** |
| | | | 0.9 | 24.62 | 21.313 | 20.457 | 18.255 | 17.198 | 18.525 | **13.003** |
| | | | 0.95 | 46.818 | 35.645 | 33.332 | 26.041 | 23.141 | 27.472 | **13.794** |
| | | | 0.99 | 224.17 | 76.794 | 72.233 | 8.0117 | 3.2652 | 28.131 | **0.973** |
| | 0.5 | 3 | 0.8 | 1.227 | 1.1362 | 1.153 | 1.0491 | 0.9706 | 1.0124 | **0.8035** |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | 0.9 | 2.2158 | 1.9183 | 1.9774 | 1.6431 | 1.4156 | 1.543 | **0.9943** |
| | | | 0.95 | 4.2136 | 3.208 | 3.4302 | 2.3436 | 1.7424 | 2.1117 | **0.8988** |
| | | | 0.99 | 20.175 | 6.9114 | 10.733 | 0.721 | 0.2033 | 0.8096 | **0.0512** |
| | | 5 | 0.8 | 3.4082 | 3.1559 | 3.2026 | 2.9139 | 2.6954 | 2.8117 | **2.2303** |
| | | | 0.9 | 6.1551 | 5.3284 | 5.4928 | 4.5638 | 3.9317 | 4.2857 | **2.7607** |
| | | | 0.95 | 11.704 | 8.9111 | 9.5284 | 6.5101 | 4.84 | 5.8658 | **2.4962** |
| | | | 0.99 | 56.042 | 19.198 | 29.813 | 2.0029 | 0.5647 | 2.2488 | **0.1419** |
| | | 10 | 0.8 | 13.633 | 12.623 | 12.81 | 11.655 | 10.78 | 11.246 | **8.9179** |
| | | | 0.9 | 24.62 | 21.313 | 21.971 | 18.255 | 15.726 | 17.142 | **11.04** |
| | | | 0.95 | 46.818 | 35.645 | 38.114 | 26.041 | 19.36 | 23.464 | **9.9845** |
| | | | 0.99 | 224.17 | 76.794 | 119.25 | 8.0117 | 2.2591 | 8.9955 | **0.5672** |
| | 0.8 | 3 | 0.8 | 1.227 | 1.1362 | 1.1971 | 1.0491 | 0.9264 | 0.9721 | **0.7377** |
| | | | 0.9 | 2.2158 | 1.9183 | 2.1188 | 1.6431 | 1.2944 | 1.4237 | **0.843** |
| | | | 0.95 | 4.2136 | 3.208 | 3.8903 | 2.3436 | 1.4547 | 1.7807 | **0.6458** |
| | | | 0.99 | 20.175 | 6.9114 | 16.04 | 0.721 | 0.4881 | 0.1629 | **0.0473** |
| | | 5 | 0.8 | 3.4082 | 3.1559 | 3.3251 | 2.9139 | 2.5725 | 2.6997 | **2.047** |
| | | | 0.9 | 6.1551 | 5.3284 | 5.8855 | 4.5638 | 3.595 | 3.9542 | **2.3401** |
| | | | 0.95 | 11.704 | 8.9111 | 10.806 | 6.5101 | 4.0406 | 4.9464 | **1.7932** |
| | | | 0.99 | 56.042 | 19.198 | 44.554 | 2.0029 | 1.3556 | 0.4524 | **0.1306** |
| | | 10 | 0.8 | 13.633 | 12.623 | 13.3 | 11.655 | 10.288 | 10.797 | **8.1834** |
| | | | 0.9 | 24.62 | 21.313 | 23.542 | 18.255 | 14.379 | 15.816 | **9.3569** |
| | | | 0.95 | 46.818 | 35.645 | 43.226 | 26.041 | 16.163 | 19.786 | **7.1724** |
| | | | 0.99 | 224.17 | 76.794 | 178.22 | 8.0117 | 5.4227 | 1.8099 | **0.522** |
| 0.9 | 0.2 | 3 | 0.8 | 1.227 | 1.0947 | 1.1098 | 0.9706 | 0.9264 | 1.0535 | **0.798** |
| | | | 0.9 | 2.2158 | 1.7923 | 1.8412 | 1.4156 | 1.2944 | 1.6674 | **0.982** |
| | | | 0.95 | 4.2136 | 2.8383 | 2.9999 | 1.7424 | 1.4547 | 2.4725 | **0.8767** |
| | | | 0.99 | 20.175 | 4.7705 | 6.5009 | 0.2033 | 0.4881 | 2.5317 | **0.089** |
| | | 5 | 0.8 | 3.4082 | 3.0407 | 3.0825 | 2.6954 | 2.5725 | 2.9261 | **2.2149** |
| | | | 0.9 | 6.1551 | 4.9784 | 5.1142 | 3.9317 | 3.595 | 4.6314 | **2.7266** |
| | | | 0.95 | 11.704 | 7.8841 | 8.333 | 4.84 | 4.0406 | 6.868 | **2.4349** |
| | | | 0.99 | 56.042 | 13.251 | 18.058 | 0.5647 | 1.3556 | 7.0326 | **0.2469** |
| | | 10 | 0.8 | 13.633 | 12.162 | 12.329 | 10.78 | 10.288 | 11.704 | **8.8563** |
| | | | 0.9 | 24.62 | 19.913 | 20.457 | 15.726 | 14.379 | 18.525 | **10.904** |
| | | | 0.95 | 46.818 | 31.537 | 33.332 | 19.36 | 16.163 | 27.472 | **9.7393** |
| | | | 0.99 | 224.17 | 53.006 | 72.233 | 2.2591 | 5.4227 | 28.131 | **0.9873** |
| | 0.5 | 3 | 0.8 | 1.227 | 1.0947 | 1.153 | 0.9706 | 0.864 | 1.0124 | **0.7167** |
| | | | 0.9 | 2.2158 | 1.7923 | 1.9774 | 1.4156 | 1.1316 | 1.543 | **0.7992** |
| | | | 0.95 | 4.2136 | 2.8383 | 3.4302 | 1.7424 | 1.1038 | 2.1117 | **0.5823** |
| | | | 0.99 | 20.175 | 4.7705 | 10.733 | 0.2033 | 1.2476 | 0.8096 | **0.0759** |
| | | 5 | 0.8 | 3.4082 | 3.0407 | 3.2026 | 2.6954 | 2.3989 | 2.8117 | **1.9884** |
| | | | 0.9 | 6.1551 | 4.9784 | 5.4928 | 3.9317 | 3.1425 | 4.2857 | **2.2182** |
| | | | 0.95 | 11.704 | 7.8841 | 9.5284 | 4.84 | 3.066 | 5.8658 | **1.6168** |
| | | | 0.99 | 56.042 | 13.251 | 29.813 | 0.5647 | 3.4652 | 2.2488 | **0.2102** |
| | | 10 | 0.8 | 13.633 | 12.162 | 12.81 | 10.78 | 9.5932 | 11.246 | **7.9487** |
| | | | 0.9 | 24.62 | 19.913 | 21.971 | 15.726 | 12.568 | 17.142 | **8.8689** |
| | | | 0.95 | 46.818 | 31.537 | 38.114 | 19.36 | 12.264 | 23.464 | **6.4666** |
| | | | 0.99 | 224.17 | 53.006 | 119.25 | 2.2591 | 13.861 | 8.9955 | **0.84** |
| | 0.8 | 3 | 0.8 | 1.227 | 1.0947 | 1.1971 | 0.9706 | 0.806 | 0.9721 | **0.6437** |
| | | | 0.9 | 2.2158 | 1.7923 | 2.1188 | 1.4156 | 0.9888 | 1.4237 | **0.6499** |
| | | | 0.95 | 4.2136 | 2.8383 | 3.8903 | 1.7424 | 0.8309 | 1.7807 | **0.3852** |
| | | | 0.99 | 20.175 | 4.7705 | 16.04 | 0.2033 | 2.1589 | 0.1629 | **0.0537** |
| | | 5 | 0.8 | 3.4082 | 3.0407 | 3.3251 | 2.6954 | 2.2373 | 2.6997 | **1.785** |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 0.9 | 6.1551 | 4.9784 | 5.8855 | 3.9317 | 2.7455 | 3.9542 | **1.8027** |
| | | 0.95 | 11.704 | 7.8841 | 10.806 | 4.84 | 2.3076 | 4.9464 | **1.0688** |
| | | 0.99 | 56.042 | 13.251 | 44.554 | 0.5647 | 5.9966 | 0.4524 | **0.148** |
| | 10 | 0.8 | 13.633 | 12.162 | 13.3 | 10.78 | 8.946 | 10.797 | **7.1334** |
| | | 0.9 | 24.62 | 19.913 | 23.542 | 15.726 | 10.979 | 15.816 | **7.2057** |
| | | 0.95 | 46.818 | 31.537 | 43.226 | 19.36 | 9.23 | 19.786 | **4.2739** |
| | | 0.99 | 224.17 | 53.006 | 178.22 | 2.2591 | 23.986 | 1.8099 | **0.5908** |

NOTE: Minimum MSE value is bolded in each row

Table 2: Estimated MSE when n=100 and p=3

| K | d | sigma | rho | OLS | RIDGE | LIU | K-L | DK | ML | BTP |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.2 | 3 | 0.8 | 0.5596 | 0.5499 | 0.5345 | 0.5403 | 0.5366 | 0.5223 | **0.501** |
| | | | 0.9 | 1.0178 | 0.9839 | 0.9322 | 0.9506 | 0.9378 | 0.8909 | **0.8214** |
| | | | 0.95 | 1.9464 | 1.8211 | 1.6447 | 1.7002 | 1.6549 | 1.5038 | **1.2811** |
| | | | 0.99 | 9.4002 | 6.9 | 4.8107 | 4.7956 | 4.1841 | 3.0985 | **1.42** |
| | | 5 | 0.8 | 1.5543 | 1.5275 | 1.4848 | 1.5009 | 1.4905 | 1.4507 | **1.3915** |
| | | | 0.9 | 2.8273 | 2.7332 | 2.5896 | 2.6407 | 2.6049 | 2.4748 | **2.2817** |
| | | | 0.95 | 5.4066 | 5.0586 | 4.5687 | 4.7227 | 4.597 | 4.1773 | **3.5585** |
| | | | 0.99 | 26.112 | 19.167 | 13.363 | 13.321 | 11.623 | 8.607 | **3.9442** |
| | | 10 | 0.8 | 6.2173 | 6.11 | 5.9393 | 6.0036 | 5.9618 | 5.8028 | **5.5657** |
| | | | 0.9 | 11.309 | 10.933 | 10.358 | 10.563 | 10.42 | 9.8992 | **9.1265** |
| | | | 0.95 | 21.626 | 20.235 | 18.275 | 18.891 | 18.388 | 16.709 | **14.234** |
| | | | 0.99 | 104.45 | 76.667 | 53.452 | 53.285 | 46.49 | 34.428 | **15.777** |
| | 0.5 | 3 | 0.8 | 0.5596 | 0.5499 | 0.5438 | 0.5403 | 0.531 | 0.5132 | **0.4872** |
| | | | 0.9 | 1.0178 | 0.9839 | 0.9639 | 0.9506 | 0.9188 | 0.8606 | **0.7778** |
| | | | 0.95 | 1.9464 | 1.8211 | 1.7548 | 1.7002 | 1.5894 | 1.4025 | **1.1488** |
| | | | 0.99 | 9.4002 | 6.9 | 6.3497 | 4.7956 | 3.4012 | 2.0693 | **0.7952** |
| | | 5 | 0.8 | 1.5543 | 1.5275 | 1.5107 | 1.5009 | 1.4749 | 1.4254 | **1.3532** |
| | | | 0.9 | 2.8273 | 2.7332 | 2.6774 | 2.6407 | 2.5523 | 2.3906 | **2.1603** |
| | | | 0.95 | 5.4066 | 5.0586 | 4.8743 | 4.7227 | 4.4148 | 3.8957 | **3.191** |
| | | | 0.99 | 26.112 | 19.167 | 17.638 | 13.321 | 9.4477 | 5.7479 | **2.2088** |
| | | 10 | 0.8 | 6.2173 | 6.11 | 6.0427 | 6.0036 | 5.8998 | 5.7016 | **5.4125** |
| | | | 0.9 | 11.309 | 10.933 | 10.71 | 10.563 | 10.209 | 9.5622 | **8.6409** |
| | | | 0.95 | 21.626 | 20.235 | 19.497 | 18.891 | 17.659 | 15.583 | **12.764** |
| | | | 0.99 | 104.45 | 76.667 | 70.552 | 53.285 | 37.791 | 22.991 | **8.8349** |
| | 0.8 | 3 | 0.8 | 0.5596 | 0.5499 | 0.5532 | 0.5403 | 0.5255 | 0.5042 | **0.4738** |
| | | | 0.9 | 1.0178 | 0.9839 | 0.9961 | 0.9506 | 0.9003 | 0.8309 | **0.7361** |
| | | | 0.95 | 1.9464 | 1.8211 | 1.8685 | 1.7002 | 1.5264 | 1.3049 | **1.0282** |
| | | | 0.99 | 9.4002 | 6.9 | 8.1072 | 4.7956 | 2.7536 | 1.2584 | **0.4166** |
| | | 5 | 0.8 | 1.5543 | 1.5275 | 1.5368 | 1.5009 | 1.4596 | 1.4004 | **1.3158** |
| | | | 0.9 | 2.8273 | 2.7332 | 2.7668 | 2.6407 | 2.5007 | 2.3079 | **2.0445** |
| | | | 0.95 | 5.4066 | 5.0586 | 5.1902 | 4.7227 | 4.24 | 3.6245 | **2.8558** |
| | | | 0.99 | 26.112 | 19.167 | 22.52 | 13.321 | 7.649 | 3.4955 | **1.157** |
| | | 10 | 0.8 | 6.2173 | 6.11 | 6.1472 | 6.0036 | 5.8384 | 5.6014 | **5.263** |
| | | | 0.9 | 11.309 | 10.933 | 11.067 | 10.563 | 10.003 | 9.2314 | **8.1776** |
| | | | 0.95 | 21.626 | 20.235 | 20.761 | 18.891 | 16.96 | 14.498 | **11.423** |
| | | | 0.99 | 104.45 | 76.667 | 90.08 | 53.285 | 30.596 | 13.982 | **4.6276** |
| 0.6 | 0.2 | 3 | 0.8 | 0.5596 | 0.5405 | 0.5345 | 0.5218 | 0.5146 | 0.5223 | **0.4806** |
| | | | 0.9 | 1.0178 | 0.9518 | 0.9322 | 0.8881 | 0.8643 | 0.8909 | **0.7576** |
| | | | 0.95 | 1.9464 | 1.708 | 1.6447 | 1.4859 | 1.408 | 1.5038 | **1.0922** |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | 0.99 | 9.4002 | 5.2929 | 4.8107 | 2.3853 | 1.775 | 3.0985 | **0.63** |
| | | 5 | 0.8 | 1.5543 | 1.5014 | 1.4848 | 1.4495 | 1.4294 | 1.4507 | **1.3348** |
| | | | 0.9 | 2.8273 | 2.6439 | 2.5896 | 2.4669 | 2.4008 | 2.4748 | **2.1043** |
| | | | 0.95 | 5.4066 | 4.7444 | 4.5687 | 4.1273 | 3.911 | 4.1773 | **3.0336** |
| | | | 0.99 | 26.112 | 14.702 | 13.363 | 6.6258 | 4.9303 | 8.607 | **1.7499** |
| | | 10 | 0.8 | 6.2173 | 6.0056 | 5.9393 | 5.7978 | 5.7176 | 5.8028 | **5.3391** |
| | | | 0.9 | 11.309 | 10.575 | 10.358 | 9.8677 | 9.6032 | 9.8992 | **8.417** |
| | | | 0.95 | 21.626 | 18.978 | 18.275 | 16.509 | 15.644 | 16.709 | **12.134** |
| | | | 0.99 | 104.45 | 58.81 | 53.452 | 26.503 | 19.721 | 34.428 | **6.9991** |
| | 0.5 | 3 | 0.8 | 0.5596 | 0.5405 | 0.5438 | 0.5218 | 0.504 | 0.5132 | **0.4627** |
| | | | 0.9 | 1.0178 | 0.9518 | 0.9639 | 0.8881 | 0.8299 | 0.8606 | **0.7033** |
| | | | 0.95 | 1.9464 | 1.708 | 1.7548 | 1.4859 | 1.2988 | 1.4025 | **0.9421** |
| | | | 0.99 | 9.4002 | 5.2929 | 6.3497 | 2.3853 | 1.1091 | 2.0693 | **0.2901** |
| | | 5 | 0.8 | 1.5543 | 1.5014 | 1.5107 | 1.4495 | 1.3999 | 1.4254 | **1.2849** |
| | | | 0.9 | 2.8273 | 2.6439 | 2.6774 | 2.4669 | 2.3051 | 2.3906 | **1.9533** |
| | | | 0.95 | 5.4066 | 4.7444 | 4.8743 | 4.1273 | 3.6077 | 3.8957 | **2.6165** |
| | | | 0.99 | 26.112 | 14.702 | 17.638 | 6.6258 | 3.0808 | 5.7479 | **0.8054** |
| | | 10 | 0.8 | 6.2173 | 6.0056 | 6.0427 | 5.7978 | 5.5996 | 5.7016 | **5.1392** |
| | | | 0.9 | 11.309 | 10.575 | 10.71 | 9.8677 | 9.2202 | 9.5622 | **7.8126** |
| | | | 0.95 | 21.626 | 18.978 | 19.497 | 16.509 | 14.431 | 15.583 | **10.465** |
| | | | 0.99 | 104.45 | 58.81 | 70.552 | 26.503 | 12.323 | 22.991 | **3.2209** |
| | 0.8 | 3 | 0.8 | 0.5596 | 0.5405 | 0.5532 | 0.5218 | 0.4936 | 0.5042 | **0.4455** |
| | | | 0.9 | 1.0178 | 0.9518 | 0.9961 | 0.8881 | 0.7968 | 0.8309 | **0.6527** |
| | | | 0.95 | 1.9464 | 1.708 | 1.8685 | 1.4859 | 1.1981 | 1.3049 | **0.8114** |
| | | | 0.99 | 9.4002 | 5.2929 | 8.1072 | 2.3853 | 0.6642 | 1.2584 | **0.1322** |
| | | 5 | 0.8 | 1.5543 | 1.5014 | 1.5368 | 1.4495 | 1.3711 | 1.4004 | **1.2368** |
| | | | 0.9 | 2.8273 | 2.6439 | 2.7668 | 2.4669 | 2.2133 | 2.3079 | **1.8126** |
| | | | 0.95 | 5.4066 | 4.7444 | 5.1902 | 4.1273 | 3.3278 | 3.6245 | **2.2533** |
| | | | 0.99 | 26.112 | 14.702 | 22.52 | 6.6258 | 1.845 | 3.4955 | **0.3668** |
| | | 10 | 0.8 | 6.2173 | 6.0056 | 6.1472 | 5.7978 | 5.4841 | 5.6014 | **4.9466** |
| | | | 0.9 | 11.309 | 10.575 | 11.067 | 9.8677 | 8.853 | 9.2314 | **7.2496** |
| | | | 0.95 | 21.626 | 18.978 | 20.761 | 16.509 | 13.311 | 14.498 | **9.0125** |
| | | | 0.99 | 104.45 | 58.81 | 90.08 | 26.503 | 7.3796 | 13.982 | **1.4662** |
| 0.9 | 0.2 | 3 | 0.8 | 0.5596 | 0.5314 | 0.5345 | 0.504 | 0.4936 | 0.5223 | **0.4612** |
| | | | 0.9 | 1.0178 | 0.9213 | 0.9322 | 0.8299 | 0.7968 | 0.8909 | **0.699** |
| | | | 0.95 | 1.9464 | 1.6055 | 1.6447 | 1.2988 | 1.1981 | 1.5038 | **0.9314** |
| | | | 0.99 | 9.4002 | 4.1956 | 4.8107 | 1.1091 | 0.6642 | 3.0985 | **0.2576** |
| | | 5 | 0.8 | 1.5543 | 1.476 | 1.4848 | 1.3999 | 1.3711 | 1.4507 | **1.2807** |
| | | | 0.9 | 2.8273 | 2.5591 | 2.5896 | 2.3051 | 2.2133 | 2.4748 | **1.9414** |
| | | | 0.95 | 5.4066 | 4.4597 | 4.5687 | 3.6077 | 3.3278 | 4.1773 | **2.5869** |
| | | | 0.99 | 26.112 | 11.654 | 13.363 | 3.0808 | 1.845 | 8.607 | **0.7152** |
| | | 10 | 0.8 | 6.2173 | 5.9041 | 5.9393 | 5.5996 | 5.4841 | 5.8028 | **5.1224** |
| | | | 0.9 | 11.309 | 10.236 | 10.358 | 9.2202 | 8.853 | 9.8992 | **7.765** |
| | | | 0.95 | 21.626 | 17.839 | 18.275 | 14.431 | 13.311 | 16.709 | **10.347** |
| | | | 0.99 | 104.45 | 46.617 | 53.452 | 12.323 | 7.3796 | 34.428 | **2.86** |
| | 0.5 | 3 | 0.8 | 0.5596 | 0.5314 | 0.5438 | 0.504 | 0.4785 | 0.5132 | **0.4396** |
| | | | 0.9 | 1.0178 | 0.9213 | 0.9639 | 0.8299 | 0.7498 | 0.8606 | **0.6363** |
| | | | 0.95 | 1.9464 | 1.6055 | 1.7548 | 1.2988 | 1.0612 | 1.4025 | **0.7727** |
| | | | 0.99 | 9.4002 | 4.1956 | 6.3497 | 1.1091 | 0.2798 | 2.0693 | **0.0972** |
| | | 5 | 0.8 | 1.5543 | 1.476 | 1.5107 | 1.3999 | 1.329 | 1.4254 | **1.2204** |
| | | | 0.9 | 2.8273 | 2.5591 | 2.6774 | 2.3051 | 2.0826 | 2.3906 | **1.7669** |
| | | | 0.95 | 5.4066 | 4.4597 | 4.8743 | 3.6077 | 2.9476 | 3.8957 | **2.1459** |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | 0.99 | 26.112 | 11.654 | 17.638 | 3.0808 | 0.777 | 5.7479 | **0.2695** |
| | | 10 | 0.8 | 6.2173 | 5.9041 | 6.0427 | 5.5996 | 5.3157 | 5.7016 | **4.8807** |
| | | | 0.9 | 11.309 | 10.236 | 10.71 | 9.2202 | 8.3302 | 9.5622 | **7.0668** |
| | | | 0.95 | 21.626 | 17.839 | 19.497 | 14.431 | 11.79 | 15.583 | **8.5828** |
| | | | 0.99 | 104.45 | 46.617 | 70.552 | 12.323 | 3.1074 | 22.991 | **1.0773** |
| | 0.8 | 3 | 0.8 | 0.5596 | 0.5314 | 0.5532 | 0.504 | 0.4639 | 0.5042 | **0.419** |
| | | | 0.9 | 1.0178 | 0.9213 | 0.9961 | 0.8299 | 0.7057 | 0.8309 | **0.5792** |
| | | | 0.95 | 1.9464 | 1.6055 | 1.8685 | 1.2988 | 0.9397 | 1.3049 | **0.6404** |
| | | | 0.99 | 9.4002 | 4.1956 | 8.1072 | 1.1091 | 0.1189 | 1.2584 | **0.0471** |
| | | 5 | 0.8 | 1.5543 | 1.476 | 1.5368 | 1.3999 | 1.2883 | 1.4004 | **1.163** |
| | | | 0.9 | 2.8273 | 2.5591 | 2.7668 | 2.3051 | 1.9599 | 2.3079 | **1.608** |
| | | | 0.95 | 5.4066 | 4.4597 | 5.1902 | 3.6077 | 2.6099 | 3.6245 | **1.7781** |
| | | | 0.99 | 26.112 | 11.654 | 22.52 | 3.0808 | 0.3301 | 3.4955 | **0.1302** |
| | | 10 | 0.8 | 6.2173 | 5.9041 | 6.1472 | 5.5996 | 5.1528 | 5.6014 | **4.6506** |
| | | | 0.9 | 11.309 | 10.236 | 11.067 | 9.2202 | 7.839 | 9.2314 | **6.4308** |
| | | | 0.95 | 21.626 | 17.839 | 20.761 | 14.431 | 10.439 | 14.498 | **7.111** |
| | | | 0.99 | 104.45 | 46.617 | 90.08 | 12.323 | 1.3197 | 13.982 | **0.5194** |

NOTE: Minimum MSE value is bolded in each row

Table 3: Estimated MSE when n=50 and p=7

| K | d | sigma | rho | OLS | RIDGE | LIU | K-L | DK | ML | BTP |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.2 | 3 | 0.8 | 3.3789 | 3.2362 | 3.0254 | 3.097 | 3.0438 | 2.8569 | **2.5803** |
| | | | 0.9 | 6.3296 | 5.8369 | 5.1772 | 5.3669 | 5.1947 | 4.6497 | **3.8479** |
| | | | 0.95 | 12.278 | 10.524 | 8.5534 | 8.9216 | 8.3799 | 6.9701 | **4.8929** |
| | | | 0.99 | 59.959 | 31.635 | 19.093 | 13.216 | 9.8364 | 7.675 | **1.9214** |
| | | 5 | 0.8 | 9.3859 | 8.9894 | 8.4038 | 8.6027 | 8.4551 | 7.9359 | **7.1673** |
| | | | 0.9 | 17.582 | 16.214 | 14.381 | 14.908 | 14.43 | 12.916 | **10.689** |
| | | | 0.95 | 34.106 | 29.232 | 23.759 | 24.782 | 23.278 | 19.361 | **13.591** |
| | | | 0.99 | 166.55 | 87.876 | 53.037 | 36.712 | 27.323 | 21.319 | **5.3373** |
| | | 10 | 0.8 | 37.544 | 35.958 | 33.615 | 34.411 | 33.82 | 31.744 | **28.669** |
| | | | 0.9 | 70.329 | 64.855 | 57.524 | 59.633 | 57.72 | 51.664 | **42.754** |
| | | | 0.95 | 136.42 | 116.93 | 95.037 | 99.129 | 93.11 | 77.444 | **54.364** |
| | | | 0.99 | 666.21 | 351.5 | 212.15 | 146.85 | 109.29 | 85.278 | **21.349** |
| | 0.5 | 3 | 0.8 | 3.3789 | 3.2362 | 3.1553 | 3.097 | 2.9661 | 2.7343 | **2.4101** |
| | | | 0.9 | 6.3296 | 5.8369 | 5.5941 | 5.3669 | 4.9479 | 4.2755 | **3.3877** |
| | | | 0.95 | 12.278 | 10.524 | 9.8629 | 8.9216 | 7.633 | 5.9046 | **3.8496** |
| | | | 0.99 | 59.959 | 31.635 | 31.601 | 13.216 | 6.3565 | 3.055 | **0.732** |
| | | 5 | 0.8 | 9.3859 | 8.9894 | 8.7648 | 8.6027 | 8.2391 | 7.5951 | **6.6946** |
| | | | 0.9 | 17.582 | 16.214 | 15.539 | 14.908 | 13.744 | 11.876 | **9.4102** |
| | | | 0.95 | 34.106 | 29.232 | 27.397 | 24.782 | 21.203 | 16.402 | **10.693** |
| | | | 0.99 | 166.55 | 87.876 | 87.781 | 36.712 | 17.657 | 8.486 | **2.0333** |
| | | 10 | 0.8 | 37.544 | 35.958 | 35.059 | 34.411 | 32.956 | 30.38 | **26.778** |
| | | | 0.9 | 70.329 | 64.855 | 62.157 | 59.633 | 54.977 | 47.505 | **37.641** |
| | | | 0.95 | 136.42 | 116.93 | 109.59 | 99.129 | 84.811 | 65.606 | **42.772** |
| | | | 0.99 | 666.21 | 351.5 | 351.12 | 146.85 | 70.628 | 33.944 | **8.1334** |
| | 0.8 | 3 | 0.8 | 3.3789 | 3.2362 | 3.2884 | 3.097 | 2.8906 | 2.6147 | **2.2505** |
| | | | 0.9 | 6.3296 | 5.8369 | 6.0293 | 5.3669 | 4.7139 | 3.9195 | **2.979** |
| | | | 0.95 | 12.278 | 10.524 | 11.277 | 8.9216 | 6.9571 | 4.9438 | **3.0171** |
| | | | 0.99 | 59.959 | 31.635 | 47.489 | 13.216 | 4.2145 | 1.8153 | **0.2557** |
| | | 5 | 0.8 | 9.3859 | 8.9894 | 9.1346 | 8.6027 | 8.0293 | 7.263 | **6.251** |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | 0.9 | 17.582 | 16.214 | 16.748 | 14.908 | 13.094 | 10.887 | **8.2749** |
| | | | 0.95 | 34.106 | 29.232 | 31.325 | 24.782 | 19.325 | 13.733 | **8.3805** |
| | | | 0.99 | 166.55 | 87.876 | 131.91 | 36.712 | 11.707 | 5.0426 | **0.7101** |
| | | 10 | 0.8 | 37.544 | 35.958 | 36.538 | 34.411 | 32.117 | 29.052 | **25.004** |
| | | | 0.9 | 70.329 | 64.855 | 66.992 | 59.633 | 52.377 | 43.55 | **33.099** |
| | | | 0.95 | 136.42 | 116.93 | 125.3 | 99.129 | 77.3 | 54.93 | **33.521** |
| | | | 0.99 | 666.21 | 351.5 | 527.66 | 146.85 | 46.828 | 20.17 | **2.8403** |
| 0.6 | 0.2 | 3 | 0.8 | 3.3789 | 3.1035 | 3.0254 | 2.8414 | 2.7459 | 2.8569 | **2.3338** |
| | | | 0.9 | 6.3296 | 5.406 | 5.1772 | 4.5647 | 4.2816 | 4.6497 | **3.1983** |
| | | | 0.95 | 12.278 | 9.1533 | 8.5534 | 6.5421 | 5.7885 | 6.9701 | **3.4784** |
| | | | 0.99 | 59.959 | 20.078 | 19.093 | 3.3132 | 2.3913 | 7.675 | **0.5193** |
| | | 5 | 0.8 | 9.3859 | 8.6207 | 8.4038 | 7.8928 | 7.6275 | 7.9359 | **6.4825** |
| | | | 0.9 | 17.582 | 15.017 | 14.381 | 12.68 | 11.893 | 12.916 | **8.8839** |
| | | | 0.95 | 34.106 | 25.426 | 23.759 | 18.172 | 16.079 | 19.361 | **9.662** |
| | | | 0.99 | 166.55 | 55.772 | 53.037 | 9.2035 | 6.6426 | 21.319 | **1.4423** |
| | | 10 | 0.8 | 37.544 | 34.483 | 33.615 | 31.571 | 30.51 | 31.744 | **25.93** |
| | | | 0.9 | 70.329 | 60.067 | 57.524 | 50.719 | 47.573 | 51.664 | **35.535** |
| | | | 0.95 | 136.42 | 101.7 | 95.037 | 72.689 | 64.316 | 77.444 | **38.647** |
| | | | 0.99 | 666.21 | 223.09 | 212.15 | 36.814 | 26.57 | 85.278 | **5.7692** |
| | 0.5 | 3 | 0.8 | 3.3789 | 3.1035 | 3.1553 | 2.8414 | 2.6094 | 2.7343 | **2.1292** |
| | | | 0.9 | 6.3296 | 5.406 | 5.5941 | 4.5647 | 3.8921 | 4.2755 | **2.7016** |
| | | | 0.95 | 12.278 | 9.1533 | 9.8629 | 6.5421 | 4.8238 | 5.9046 | **2.5544** |
| | | | 0.99 | 59.959 | 20.078 | 31.601 | 3.3132 | 2.4148 | 3.055 | **0.1571** |
| | | 5 | 0.8 | 9.3859 | 8.6207 | 8.7648 | 7.8928 | 7.2481 | 7.5951 | **5.9139** |
| | | | 0.9 | 17.582 | 15.017 | 15.539 | 12.68 | 10.811 | 11.876 | **7.5043** |
| | | | 0.95 | 34.106 | 25.426 | 27.397 | 18.172 | 13.399 | 16.402 | **7.095** |
| | | | 0.99 | 166.55 | 55.772 | 87.781 | 9.2035 | 6.7076 | 8.486 | **0.4363** |
| | | 10 | 0.8 | 37.544 | 34.483 | 35.059 | 31.571 | 28.992 | 30.38 | **23.655** |
| | | | 0.9 | 70.329 | 60.067 | 62.157 | 50.719 | 43.245 | 47.505 | **30.017** |
| | | | 0.95 | 136.42 | 101.7 | 109.59 | 72.689 | 53.597 | 65.606 | **28.379** |
| | | | 0.99 | 666.21 | 223.09 | 351.12 | 36.814 | 26.831 | 33.944 | **1.7452** |
| | 0.8 | 3 | 0.8 | 3.3789 | 3.1035 | 3.2884 | 2.8414 | 2.4803 | 2.6147 | **1.9431** |
| | | | 0.9 | 6.3296 | 5.406 | 6.0293 | 4.5647 | 3.5405 | 3.9195 | **2.2842** |
| | | | 0.95 | 12.278 | 9.1533 | 11.277 | 6.5421 | 4.0243 | 4.9438 | **1.8831** |
| | | | 0.99 | 59.959 | 20.078 | 47.489 | 3.3132 | 3.3704 | 1.8153 | **0.1319** |
| | | 5 | 0.8 | 9.3859 | 8.6207 | 9.1346 | 7.8928 | 6.8897 | 7.263 | **5.3967** |
| | | | 0.9 | 17.582 | 15.017 | 16.748 | 12.68 | 9.8346 | 10.887 | **6.3445** |
| | | | 0.95 | 34.106 | 25.426 | 31.325 | 18.172 | 11.178 | 13.733 | **5.2301** |
| | | | 0.99 | 166.55 | 55.772 | 131.91 | 9.2035 | 9.3621 | 5.0426 | **0.3661** |
| | | 10 | 0.8 | 37.544 | 34.483 | 36.538 | 31.571 | 27.558 | 29.052 | **21.586** |
| | | | 0.9 | 70.329 | 60.067 | 66.992 | 50.719 | 39.338 | 43.55 | **25.378** |
| | | | 0.95 | 136.42 | 101.7 | 125.3 | 72.689 | 44.713 | 54.93 | **20.919** |
| | | | 0.99 | 666.21 | 223.09 | 527.66 | 36.814 | 37.448 | 20.17 | **1.464** |
| 0.9 | 0.2 | 3 | 0.8 | 3.3789 | 2.9798 | 3.0254 | 2.6094 | 2.4803 | 2.8569 | **2.1137** |
| | | | 0.9 | 6.3296 | 5.0261 | 5.1772 | 3.8921 | 3.5405 | 4.6497 | **2.6674** |
| | | | 0.95 | 12.278 | 8.0565 | 8.5534 | 4.8238 | 4.0243 | 6.9701 | **2.492** |
| | | | 0.99 | 59.959 | 14.03 | 19.093 | 2.4148 | 3.3704 | 7.675 | **0.2829** |
| | | 5 | 0.8 | 9.3859 | 8.2772 | 8.4038 | 7.2481 | 6.8897 | 7.9359 | **5.8708** |
| | | | 0.9 | 17.582 | 13.962 | 14.381 | 10.811 | 9.8346 | 12.916 | **7.4091** |
| | | | 0.95 | 34.106 | 22.379 | 23.759 | 13.399 | 11.178 | 19.361 | **6.9217** |
| | | | 0.99 | 166.55 | 38.973 | 53.037 | 6.7076 | 9.3621 | 21.319 | **0.7855** |
| | | 10 | 0.8 | 37.544 | 33.109 | 33.615 | 28.992 | 27.558 | 31.744 | **23.482** |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | 0.9 | 70.329 | 55.846 | 57.524 | 43.245 | 39.338 | 51.664 | **29.636** |
| | | | 0.95 | 136.42 | 89.516 | 95.037 | 53.597 | 44.713 | 77.444 | **27.686** |
| | | | 0.99 | 666.21 | 155.89 | 212.15 | 26.831 | 37.448 | 85.278 | **3.142** |
| | 0.5 | 3 | 0.8 | 3.3789 | 2.9798 | 3.1553 | 2.6094 | 2.2999 | 2.7343 | **1.8846** |
| | | | 0.9 | 6.3296 | 5.0261 | 5.5941 | 3.8921 | 3.0752 | 4.2755 | **2.1649** |
| | | | 0.95 | 12.278 | 8.0565 | 9.8629 | 4.8238 | 3.0709 | 5.9046 | **1.7116** |
| | | | 0.99 | 59.959 | 14.03 | 31.601 | 2.4148 | 5.6014 | 3.055 | **0.0737** |
| | | 5 | 0.8 | 9.3859 | 8.2772 | 8.7648 | 7.2481 | 6.3883 | 7.5951 | **5.2342** |
| | | | 0.9 | 17.582 | 13.962 | 15.539 | 10.811 | 8.5421 | 11.876 | **6.013** |
| | | | 0.95 | 34.106 | 22.379 | 27.397 | 13.399 | 8.53 | 16.402 | **4.7538** |
| | | | 0.99 | 166.55 | 38.973 | 87.781 | 6.7076 | 15.559 | 8.486 | **0.2044** |
| | | 10 | 0.8 | 37.544 | 33.109 | 35.059 | 28.992 | 25.553 | 30.38 | **20.936** |
| | | | 0.9 | 70.329 | 55.846 | 62.157 | 43.245 | 34.168 | 47.505 | **24.052** |
| | | | 0.95 | 136.42 | 89.516 | 109.59 | 53.597 | 34.119 | 65.606 | **19.014** |
| | | | 0.99 | 666.21 | 155.89 | 351.12 | 26.831 | 62.237 | 33.944 | **0.8175** |
| | 0.8 | 3 | 0.8 | 3.3789 | 2.9798 | 3.2884 | 2.6094 | 2.1338 | 2.6147 | **1.6822** |
| | | | 0.9 | 6.3296 | 5.0261 | 6.0293 | 3.8921 | 2.6741 | 3.9195 | **1.7624** |
| | | | 0.95 | 12.278 | 8.0565 | 11.277 | 4.8238 | 2.3469 | 4.9438 | **1.1892** |
| | | | 0.99 | 59.959 | 14.03 | 47.489 | 2.4148 | 8.1754 | 1.8153 | **0.2111** |
| | | 5 | 0.8 | 9.3859 | 8.2772 | 9.1346 | 7.2481 | 5.9267 | 7.263 | **4.6714** |
| | | | 0.9 | 17.582 | 13.962 | 16.748 | 10.811 | 7.4278 | 10.887 | **4.8948** |
| | | | 0.95 | 34.106 | 22.379 | 31.325 | 13.399 | 6.5188 | 13.733 | **3.3022** |
| | | | 0.99 | 166.55 | 38.973 | 131.91 | 6.7076 | 22.709 | 5.0426 | **0.5859** |
| | | 10 | 0.8 | 37.544 | 33.109 | 36.538 | 28.992 | 23.706 | 29.052 | **18.684** |
| | | | 0.9 | 70.329 | 55.846 | 66.992 | 43.245 | 29.711 | 43.55 | **19.578** |
| | | | 0.95 | 136.42 | 89.516 | 125.3 | 53.597 | 26.074 | 54.93 | **13.207** |
| | | | 0.99 | 666.21 | 155.89 | 527.66 | 26.831 | 90.837 | 20.17 | **2.343** |

NOTE: Minimum MSE value is bolded in each row.

Table 4: Estimated MSE when n=100 and p=7

| K | d | sigma | rho | OLS | RIDGE | LIU | K-L | DK | ML | BTP |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.2 | 3 | 0.8 | 1.5582 | 1.5296 | 1.4842 | 1.5013 | 1.4901 | 1.448 | **1.3851** |
| | | | 0.9 | 2.9335 | 2.8321 | 2.6781 | 2.7326 | 2.6942 | 2.555 | **2.3483** |
| | | | 0.95 | 5.7032 | 5.3277 | 4.8022 | 4.9658 | 4.8307 | 4.3823 | **3.7215** |
| | | | 0.99 | 27.908 | 20.44 | 14.275 | 14.191 | 12.39 | 9.2139 | **4.2782** |
| | | 5 | 0.8 | 4.3282 | 4.2488 | 4.1228 | 4.1701 | 4.1393 | 4.0221 | **3.8474** |
| | | | 0.9 | 8.1487 | 7.867 | 7.4391 | 7.5906 | 7.484 | 7.0971 | **6.5231** |
| | | | 0.95 | 15.842 | 14.799 | 13.34 | 13.794 | 13.419 | 12.173 | **10.337** |
| | | | 0.99 | 77.524 | 56.777 | 39.653 | 39.418 | 34.416 | 25.594 | **11.884** |
| | | 10 | 0.8 | 17.313 | 16.995 | 16.491 | 16.681 | 16.557 | 16.088 | **15.39** |
| | | | 0.9 | 32.595 | 31.468 | 29.756 | 30.363 | 29.936 | 28.389 | **26.092** |
| | | | 0.95 | 63.369 | 59.197 | 53.358 | 55.175 | 53.674 | 48.693 | **41.35** |
| | | | 0.99 | 310.09 | 227.11 | 158.61 | 157.67 | 137.66 | 102.38 | **47.535** |
| | 0.5 | 3 | 0.8 | 1.5582 | 1.5296 | 1.5117 | 1.5013 | 1.4736 | 1.4211 | **1.3445** |
| | | | 0.9 | 2.9335 | 2.8321 | 2.7724 | 2.7326 | 2.6377 | 2.4647 | **2.2188** |
| | | | 0.95 | 5.7032 | 5.3277 | 5.1305 | 4.9658 | 4.6351 | 4.0808 | **3.3305** |
| | | | 0.99 | 27.908 | 20.44 | 18.839 | 14.191 | 10.09 | 6.1861 | **2.4333** |
| | | 5 | 0.8 | 4.3282 | 4.2488 | 4.1992 | 4.1701 | 4.0934 | 3.9474 | **3.7347** |
| | | | 0.9 | 8.1487 | 7.867 | 7.7012 | 7.5906 | 7.3269 | 6.8463 | **6.1632** |
| | | | 0.95 | 15.842 | 14.799 | 14.251 | 13.794 | 12.875 | 11.336 | **9.2513** |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | 0.99 | 77.524 | 56.777 | 52.33 | 39.418 | 28.028 | 17.184 | **6.759** |
| | | 10 | 0.8 | 17.313 | 16.995 | 16.797 | 16.681 | 16.374 | 15.79 | **14.939** |
| | | | 0.9 | 32.595 | 31.468 | 30.805 | 30.363 | 29.308 | 27.385 | **24.653** |
| | | | 0.95 | 63.369 | 59.197 | 57.006 | 55.175 | 51.501 | 45.342 | **37.005** |
| | | | 0.99 | 310.09 | 227.11 | 209.32 | 157.67 | 112.11 | 68.735 | **27.036** |
| | 0.8 | 3 | 0.8 | 1.5582 | 1.5296 | 1.5395 | 1.5013 | 1.4573 | 1.3945 | **1.305** |
| | | | 0.9 | 2.9335 | 2.8321 | 2.8685 | 2.7326 | 2.5824 | 2.3761 | **2.0954** |
| | | | 0.95 | 5.7032 | 5.3277 | 5.4703 | 4.9658 | 4.4477 | 3.7908 | **2.9748** |
| | | | 0.99 | 27.908 | 20.44 | 24.061 | 14.191 | 8.1931 | 3.8168 | **1.3008** |
| | | 5 | 0.8 | 4.3282 | 4.2488 | 4.2764 | 4.1701 | 4.0481 | 3.8735 | **3.6248** |
| | | | 0.9 | 8.1487 | 7.867 | 7.9681 | 7.5906 | 7.1733 | 6.6003 | **5.8204** |
| | | | 0.95 | 15.842 | 14.799 | 15.195 | 13.794 | 12.355 | 10.53 | **8.2633** |
| | | | 0.99 | 77.524 | 56.777 | 66.837 | 39.418 | 22.759 | 10.602 | **3.6132** |
| | | 10 | 0.8 | 17.313 | 16.995 | 17.105 | 16.681 | 16.192 | 15.494 | **14.499** |
| | | | 0.9 | 32.595 | 31.468 | 31.872 | 30.363 | 28.693 | 26.401 | **23.282** |
| | | | 0.95 | 63.369 | 59.197 | 60.781 | 55.175 | 49.419 | 42.12 | **33.053** |
| | | | 0.99 | 310.09 | 227.11 | 267.35 | 157.67 | 91.035 | 42.409 | **14.453** |
| 0.6 | 0.2 | 3 | 0.8 | 1.5582 | 1.5018 | 1.4842 | 1.4466 | 1.4253 | 1.448 | **1.3251** |
| | | | 0.9 | 2.9335 | 2.7362 | 2.6781 | 2.5462 | 2.4754 | 2.555 | **2.1592** |
| | | | 0.95 | 5.7032 | 4.99 | 4.8022 | 4.3272 | 4.0959 | 4.3823 | **3.1636** |
| | | | 0.99 | 27.908 | 15.687 | 14.275 | 7.1158 | 5.3331 | 9.2139 | **1.9372** |
| | | 5 | 0.8 | 4.3282 | 4.1717 | 4.1228 | 4.0182 | 3.9591 | 4.0221 | **3.6808** |
| | | | 0.9 | 8.1487 | 7.6006 | 7.4391 | 7.0728 | 6.8761 | 7.0971 | **5.9978** |
| | | | 0.95 | 15.842 | 13.861 | 13.34 | 12.02 | 11.377 | 12.173 | **8.7878** |
| | | | 0.99 | 77.524 | 43.574 | 39.653 | 19.766 | 14.814 | 25.594 | **5.381** |
| | | 10 | 0.8 | 17.313 | 16.687 | 16.491 | 16.073 | 15.836 | 16.088 | **14.723** |
| | | | 0.9 | 32.595 | 30.402 | 29.756 | 28.291 | 27.504 | 28.389 | **23.991** |
| | | | 0.95 | 63.369 | 55.444 | 53.358 | 48.079 | 45.51 | 48.693 | **35.151** |
| | | | 0.99 | 310.09 | 174.3 | 158.61 | 79.065 | 59.257 | 102.38 | **21.524** |
| | 0.5 | 3 | 0.8 | 1.5582 | 1.5018 | 1.5117 | 1.4466 | 1.394 | 1.4211 | **1.2723** |
| | | | 0.9 | 2.9335 | 2.7362 | 2.7724 | 2.5462 | 2.373 | 2.4647 | **1.9985** |
| | | | 0.95 | 5.7032 | 4.99 | 5.1305 | 4.3272 | 3.7723 | 4.0808 | **2.7219** |
| | | | 0.99 | 27.908 | 15.687 | 18.839 | 7.1158 | 3.3905 | 6.1861 | **0.9046** |
| | | 5 | 0.8 | 4.3282 | 4.1717 | 4.1992 | 4.0182 | 3.8721 | 3.9474 | **3.5341** |
| | | | 0.9 | 8.1487 | 7.6006 | 7.7012 | 7.0728 | 6.5917 | 6.8463 | **5.5514** |
| | | | 0.95 | 15.842 | 13.861 | 14.251 | 12.02 | 10.479 | 11.336 | **7.5609** |
| | | | 0.99 | 77.524 | 43.574 | 52.33 | 19.766 | 9.4181 | 17.184 | **2.5127** |
| | | 10 | 0.8 | 17.313 | 16.687 | 16.797 | 16.073 | 15.488 | 15.79 | **14.136** |
| | | | 0.9 | 32.595 | 30.402 | 30.805 | 28.291 | 26.367 | 27.385 | **22.205** |
| | | | 0.95 | 63.369 | 55.444 | 57.006 | 48.079 | 41.914 | 45.342 | **30.243** |
| | | | 0.99 | 310.09 | 174.3 | 209.32 | 79.065 | 37.672 | 68.735 | **10.051** |
| | 0.8 | 3 | 0.8 | 1.5582 | 1.5018 | 1.5395 | 1.4466 | 1.3634 | 1.3945 | **1.2216** |
| | | | 0.9 | 2.9335 | 2.7362 | 2.8685 | 2.5462 | 2.275 | 2.3761 | **1.8492** |
| | | | 0.95 | 5.7032 | 4.99 | 5.4703 | 4.3272 | 3.4744 | 3.7908 | **2.3385** |
| | | | 0.99 | 27.908 | 15.687 | 24.061 | 7.1158 | 2.0921 | 3.8168 | **0.4009** |
| | | 5 | 0.8 | 4.3282 | 4.1717 | 4.2764 | 4.0182 | 3.7872 | 3.8735 | **3.393** |
| | | | 0.9 | 8.1487 | 7.6006 | 7.9681 | 7.0728 | 6.3194 | 6.6003 | **5.1366** |
| | | | 0.95 | 15.842 | 13.861 | 15.195 | 12.02 | 9.6511 | 10.53 | **6.4957** |
| | | | 0.99 | 77.524 | 43.574 | 66.837 | 19.766 | 5.8112 | 10.602 | **1.1136** |
| | | 10 | 0.8 | 17.313 | 16.687 | 17.105 | 16.073 | 15.149 | 15.494 | **13.572** |
| | | | 0.9 | 32.595 | 30.402 | 31.872 | 28.291 | 25.278 | 26.401 | **20.546** |
| | | | 0.95 | 63.369 | 55.444 | 60.781 | 48.079 | 38.604 | 42.12 | **25.983** |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | 0.99 | 310.09 | 174.3 | 267.35 | 79.065 | 23.245 | 42.409 | **4.4544** |
| 0.9 | 0.2 | 3 | 0.8 | 1.5582 | 1.4748 | 1.4842 | 1.394 | 1.3634 | 1.448 | **1.2679** |
| | | | 0.9 | 2.9335 | 2.6454 | 2.6781 | 2.373 | 2.275 | 2.555 | **1.9859** |
| | | | 0.95 | 5.7032 | 4.6849 | 4.8022 | 3.7723 | 3.4744 | 4.3823 | **2.6906** |
| | | | 0.99 | 27.908 | 12.451 | 14.275 | 3.3905 | 2.0921 | 9.2139 | **0.8106** |
| | | 5 | 0.8 | 4.3282 | 4.0968 | 4.1228 | 3.8721 | 3.7872 | 4.0221 | **3.5218** |
| | | | 0.9 | 8.1487 | 7.3482 | 7.4391 | 6.5917 | 6.3194 | 7.0971 | **5.5164** |
| | | | 0.95 | 15.842 | 13.014 | 13.34 | 10.479 | 9.6511 | 12.173 | **7.4739** |
| | | | 0.99 | 77.524 | 34.587 | 39.653 | 9.4181 | 5.8112 | 25.594 | **2.2515** |
| | | 10 | 0.8 | 17.313 | 16.387 | 16.491 | 15.488 | 15.149 | 16.088 | **14.087** |
| | | | 0.9 | 32.595 | 29.393 | 29.756 | 26.367 | 25.278 | 28.389 | **22.065** |
| | | | 0.95 | 63.369 | 52.055 | 53.358 | 41.914 | 38.604 | 48.693 | **29.896** |
| | | | 0.99 | 310.09 | 138.35 | 158.61 | 37.672 | 23.245 | 102.38 | **9.0059** |
| | 0.5 | 3 | 0.8 | 1.5582 | 1.4748 | 1.5117 | 1.394 | 1.3189 | 1.4211 | **1.2042** |
| | | | 0.9 | 2.9335 | 2.6454 | 2.7724 | 2.373 | 2.1358 | 2.4647 | **1.8009** |
| | | | 0.95 | 5.7032 | 4.6849 | 5.1305 | 3.7723 | 3.0708 | 4.0808 | **2.2253** |
| | | | 0.99 | 27.908 | 12.451 | 18.839 | 3.3905 | 0.9662 | 6.1861 | **0.291** |
| | | 5 | 0.8 | 4.3282 | 4.0968 | 4.1992 | 3.8721 | 3.6634 | 3.9474 | **3.3449** |
| | | | 0.9 | 8.1487 | 7.3482 | 7.7012 | 6.5917 | 5.9326 | 6.8463 | **5.0023** |
| | | | 0.95 | 15.842 | 13.014 | 14.251 | 10.479 | 8.53 | 11.336 | **6.1812** |
| | | | 0.99 | 77.524 | 34.587 | 52.33 | 9.4181 | 2.684 | 17.184 | **0.8083** |
| | | 10 | 0.8 | 17.313 | 16.387 | 16.797 | 15.488 | 14.654 | 15.79 | **13.379** |
| | | | 0.9 | 32.595 | 29.393 | 30.805 | 26.367 | 23.73 | 27.385 | **20.009** |
| | | | 0.95 | 63.369 | 52.055 | 57.006 | 41.914 | 34.12 | 45.342 | **24.725** |
| | | | 0.99 | 310.09 | 138.35 | 209.32 | 37.672 | 10.736 | 68.735 | **3.2329** |
| | 0.8 | 3 | 0.8 | 1.5582 | 1.4748 | 1.5395 | 1.394 | 1.2759 | 1.3945 | **1.1438** |
| | | | 0.9 | 2.9335 | 2.6454 | 2.8685 | 2.373 | 2.0052 | 2.3761 | **1.6328** |
| | | | 0.95 | 5.7032 | 4.6849 | 5.4703 | 3.7723 | 2.7132 | 3.7908 | **1.838** |
| | | | 0.99 | 27.908 | 12.451 | 24.061 | 3.3905 | 0.4889 | 3.8168 | **0.1014** |
| | | 5 | 0.8 | 4.3282 | 4.0968 | 4.2764 | 3.8721 | 3.5439 | 3.8735 | **3.1768** |
| | | | 0.9 | 8.1487 | 7.3482 | 7.9681 | 6.5917 | 5.5699 | 6.6003 | **4.5354** |
| | | | 0.95 | 15.842 | 13.014 | 15.195 | 10.479 | 7.5367 | 10.53 | **5.1055** |
| | | | 0.99 | 77.524 | 34.587 | 66.837 | 9.4181 | 1.3581 | 10.602 | **0.2815** |
| | | 10 | 0.8 | 17.313 | 16.387 | 17.105 | 15.488 | 14.176 | 15.494 | **12.707** |
| | | | 0.9 | 32.595 | 29.393 | 31.872 | 26.367 | 22.279 | 26.401 | **18.141** |
| | | | 0.95 | 63.369 | 52.055 | 60.781 | 41.914 | 30.147 | 42.12 | **20.422** |
| | | | 0.99 | 310.09 | 138.35 | 267.35 | 37.672 | 5.4322 | 42.409 | **1.1258** |

NOTE: Minimum MSE value is bolded in each row.

Simulation results discussions

Table 1-4 shows that when σ, ρ, and p increase, the estimated MSE values also increase. In addition, the MSE decreases as the sample size *n* increase. The OLS estimator performs the worst among all the estimators used in this study which is expected due to multicollinearity. Also, we observe that the proposed BTP estimator dominates other estimators in this study. Thus, the findings agree with the theoretical results. For each row, the smallest MSE value is bolded.

## 5. Application

In this section, Portland cement data was used to demonstrate the performance of the proposed estimator. The Portland cement data was originally adopted by Woods *et al*. [32] and was later adopted by other authors such as Kaciranlar *et al.* [20], Ayinde *et al*. [33], and Lukman *et al*. [34]. The data set is widely known as the Portland cement

Table 5. Results of regression coefficients and the MSE values for Portland Cement Data

| Coef | OLS | RIDGE | LIU | KL | ML | DK | BTP |
|---|---|---|---|---|---|---|---|
| $\hat{\beta}_1$ | -52.9936 | -50.5173 | -50.9877 | -48.041 | -46.9760 | 42.3386 | -45.7216 |
| $\hat{\beta}_2$ | 0.071073 | 0.0706 | 0.0707 | 0.0700 | 0.0698 | 43.9155 | 0.0696 |
| $\hat{\beta}_3$ | -0.4142 | -0.41 | -0.4107 | -0.4056 | -0.4037 | 12.7444 | -0.4015 |
| $\hat{\beta}_4$ | -0.42347 | -0.4295 | -0.4283 | -0.4354 | -0.4380 | 0.8349 | -0.4411 |
| $\hat{\beta}_5$ | -0.57257 | -0.574 | -0.5737 | -0.5754 | -0.576 | 0.3622 | -0.5767 |
| $\hat{\beta}_6$ | 48.41787 | 48.1434 | 48.1951 | 47.869 | 47.75 | -4.8196 | 47.6109 |
| MSE | 17095.15 | 15669.12 | 15934.2 | 14318.66 | 13716.23 | 13122.63 | **10605.56** |
| Kld | | 0.6 | 0.5 | 0.6 | 0.5 | 0.6/0.5 | 0.6/0.5 |

dataset. The regression model for these data is defined as follows:

$$y_i = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \varepsilon_i \tag{47}$$

where $y_i$ = heat evolved after 180 days of curing measured in calories per gram of cement, $X_1$ = tricalcium aluminate, $X_2$ = tricalcium silicate, $X_3$ = tetracalcium aluminoferrite, and $X_4 = \beta$-dicalcium silicate. The variance inflation factors are VIF1 = 38.50, VIF2 = 254.42, VIF3 = 46.87, and VIF4 = 282.51. Eigenvalues of $X'X$ matrix are $\lambda 1$ = 44676.206, $\lambda 2$ = 5965.422, $\lambda 3$ = 809.952, and $\lambda 4$ = 105.419, and the condition number of $X'X$ is approximately 424. The VIFs, eigenvalues, and condition numbers indicate that severe multicollinearity exists. The estimated parameters and the MSE values of the estimators are presented in Table 5. It is clearly shown and obvious from Table 5 that the proposed BTP estimator dominates all other considered estimators in this study.

## 6. Conclusion

This paper proposes a new biased regression estimator called BTP as an alternative to OLS to address the problem of multicollinearity in the linear regression model. We theoretically obtained the biasing parameters of the proposed and compared this new estimation method with some existing estimators (OLS, ORR, LIU, KL, ML, and DK). The simulation study and the theoretical findings revealed the superiority of the proposed method. Likewise, real-life data was used and analyzed to support the theoretical finding and simulation results. Furthermore, it can be seen that the performance of the estimators depends on the biasing parameter.

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## References

[1] D. N. Gujarati, *Basic Econometrics*, McGraw-Hill, New York, NY, USA (1995).
[2] A. F. Lukman, *Classification-Based Ridge*, Lambert Academic Publishing (2018).
[3] K. Ayinde, A. F. Lukman, O. O. Alabi & H, A. Bello, "A New Approach of Principal Component Regression Estimator with Applications to Collinear Data", International Journal of Engineering Research and Technology **13** (2020).
[4] C. Stein, "Inadmissibility of the Usual Estimator for the Mean of a Multivariate Normal Distribution. Proceedings of the Third Berkeley Symposium on Mathematical Statistics and Probability", University California Press (1956) 197.
[5] W. F. Massy, "Principal components regression in exploratory statistical research", Journal of the American Statistical Association **60** (1965) 234.
[6] H. Wold, *Partial least squares*, In Kotz, Samuel; Johnson, Norman L. (eds.)", Encyclopedia of statistical sciences, New York: Wiley **6** (1985) 581.
[7] A. E. Hoerl & R. W. Kennard, "Ridge regression: biased estimation for nonorthogonal problems", Technometrics **12** (1970) 55.
[8] K. Liu, "A new class of biased estimate in linear regression", Communications in Statistics- Theory and Methods **22** (1993) 393.
[9] B. Swindel, "Good ridge estimators based on prior information", Communications in Statistics - Theory and Methods **A5** (1976) 1065.

[10] M. R. Özkale & S. Kaçiranlar, "The restricted and unrestricted two-parameter estimators", Communications in Statistics - Theory and Methods **36** (2007) 2707.
[11] B. M. G. Kibria & A. F. Lukman, "A New Ridge-Type Estimator for the Linear Regression Model: Simulations and Applications", Scientifica (2020) 1.
[12] I. Dawoud & B. M. G. Kibria, "A New Biased Estimator to Combat the Multicollinearity of the Gaussian Linear Regression Model", Stats **3** (2020) 526.
[13] A. T. Owolabi, K. Ayinde, J. I. Idowu, O. J. Oladapo & A. F. Lukman, "A New Two-Parameter Estimator in the Linear Regression Model with Correlated Regressors", Journal of Statistics Applications & Probability **11** (2022) 185.
[14] O. J. Oladapo, A. T. Owolabi, J. I. Idowu & K. Ayinde, "A New Modified Liu Ridge-Type Estimator for the Linear Regression Model: Simulation and Application", Int J Clin Biostat Biom. **8** (2022) 048.
[15] I. Dawoud, M. R. Abonazel & E. T. Eldin, "Predictive Performance Evaluation of the Kibria-Lukman Estimator", WSEAS TRANSACTIONS on MATHEMATICS **21** (2022) 2224.
[16] I. Dawoud, M. R. Abonazel & F. A. Awwad, "Generalized Kibria-Lukman Estimator: Method, Simulation, and Application", Front. Appl. Math. Stat. **8** (2022) 880086.
[17] I. Dawoud, M. R. Abonazel, F. A. Awwad & E. Tag Eldin, "A New Tobit Ridge-Type Estimator of the Censored Regression Model With Multicollinearity Problem", Front. Appl. Math. Stat. **8** (2022) 952142.
[18] A. F. Lukman, B. M. G. Kibria, K. Ayinde & S. L. Jegede, "Modified One-Parameter Liu Estimator for the Linear Regression Model", Modelling and Simulation in Engineering **2020** (2020) 9574304.
[19] K. Liu, "Using Liu Type Estimator to Combat Collinearity", Communications in Statistics-Theory and Methods **32** (2003) 1009.
[20] S. Kaciranlar, S. Sakallioglu, F. Akdeniz, G. P. H. Styan & H. J. Werner, "A new biased estimator in linear regression and a detailed analysis of the widely-analysed dataset on Portland Cement", Sankhya: Indian Journal of Statistics **61** (1999) 443.
[21] R. Farebrother, "Further results on the mean square error of ridge regression", J R Stat Soc. **B38** (1976) 248.
[22] G. Trenkler & H. Toutenburg, "Mean squared error matrix comparisons between biased estimators-an overview of recent results", Stat Pap. **31** (1990) 165.
[23] A. F. Lukman & K. Ayinde, "Review and classifications of the ridge parameter estimation techniques", Hacettepe Math Stat **46** (2017) 953.
[24] K. Ayinde, A. F. Lukman, O. O. Samuel & S. Ajiboye, "Some new adjusted ridge estimators of linear regression model", Int J Civ Eng Technol **9** (2018) 2838.
[25] A. T. Owolabi, K. Ayinde & O. O. Alabi, "A New Ridge-Type Estimator for the Linear Regression Model with correlated regressors", Concurrency and Computation: Practice and Experience (2022) CPE6933.
[26] I. Dawoud, A. Lukman & H. Abdul-Rahaman, "A new biased regression estimator: Theory, simulation, and application", Scientific African (2022).
[27] G. C. McDonald & D. I. Galarneau, "A Monte Carlo evaluation of some ridge-type estimators", Journal of the American Statistical Association **70** (1975) 407.
[28] D. G. Gibbons, "A simulation study of some ridge estimators", Journal of the American Statistical Association **76** (1981) 131.
[29] A. F. Lukman, K. Ayinde, S. K. Sek & E. Adewuyi, "A modified new two-parameter estimator in a linear regression model", Model. Simul. Eng. (2019) 1.
[30] L. L. Firinguetti, "A simulation study of ridge regression estimators with autocorrelated errors", Communications in Statistics Simulation and Computation **18** (1989) 673.
[31] J. P. Newhouse & S. D. Oman,"An evaluation of ridge estimators", A report prepared for United States air force project RAND (1971).
[32] H. Woods, H. H. Steinour & Starke, "Effect of composition of Portland cement on heat evolved during hardening", Industrial & Engineering Chemistry **24** (1932) 1207.
[33] K. Ayinde, A. F. Lukman, O. O. Samuel & S. A. Ajiboye, "Some new adjusted ridge estimators of linear regression model", Int. J. Civ. Eng. Technol. **9** (2018) 2838.
[34] A. F. Lukman, K. Ayinde, S. Binuomote & O. A. Clement, "Modified ridge-type estimator to combat multicollinearity: application to chemical data", Journal of Chemometrics **33** (2019b) e3125.