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On the biased Two-Parameter Estimator to Combat Multicollinearity in Linear Regression Model

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Abstract

The most popularly used estimator to estimate the regression parameters in the linear regression model is the ordinary least-squares (OLS). The existence of multicollinearity in the model renders OLS inefficient. To overcome the multicollinearity problem, a new two-parameter estimator, a biased two-parameter (BTP), is proposed as an alternative to the OLS. Theoretical comparisons and simulation studies were carried out. The theoretical comparison and simulation studies show that the proposed estimator dominated some existing estimators using the mean square error (MSE) criterion. Furthermore, the real-life data bolster both the hypothetical and simulation results. The proposed estimator is preferred to OLS and other existing estimators when multicollinearity is present in the model.

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1. Introduction

Consider a linear regression model

$$y = X\beta + \varepsilon \tag{1}$$

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where y is a $n \times 1$ vector of the dependent variable, X is a known $n \times p$ full rank matrix of explanatory variables, β is a $p \times 1$ vector of regression coefficients and $\varepsilon \sim N\left(0, \sigma^2\right)$. The ordinary least squares estimator (OLS) of β in model 1 is defined as:

$$\hat{\beta}_{OLS} = (X'X)^{-1} X'y \tag{2}$$

The Ordinary Least Squares (OLS) estimator is the best linear unbiased estimator possessing minimum variance if the usual assumptions of a classical linear regression model are satisfied. Gujarati [1], Lukman [2], Ayinde et al. [3]. However, when the column vectors of the regressors are linearly dependent, known as multicollinearity, the OLSE becomes unstable. OLS produces unbiased but inefficient estimates of the model parameters [3].

Biased estimators such as Stein estimator [4], Principal Component estimator [5], Partial Least Square estimator [6], Ridge regression estimator [7], Liu estimator [8], modified ridge regression estimator [9], two-parameter (TP) estimator [10], KL estimator [11], DK estimator [12], proposed two-parameter estimator [13], Liu-Dawoud-Kibria estimator [14], New ridge-type estimator [15], Generalized Kibria-Lukman estimator [16], Tobit new ridge-type [17] and others are often used to address this problem.

The objective of this paper is to introduce a new two-parameter estimator to address the problem of multicollinearity in linear regression and to compare the performance of the new estimator with the OLS estimator, the ordinary ridge regression (ORR) estimator [7], the Liu estimator [8], the modified one-parameter Liu (ML) estimator [18] and the Dawoud-Kibria (DK) estimator [12]. Then, below is a short description of each estimator to compare with the new estimator.

The canonical form of (1) can be written as:

$$y = Z\alpha + u \tag{3}$$

where Z=XQ, $\alpha = Q'\beta$ and Q is the orthogonal matrix such that $Z'Z = Q'X'XQ = \Lambda = diag\left(\lambda_1, \lambda_2, ..., \lambda_p\right)$. The OLS estimator of $\alpha$is given as follows:

$$\hat{\alpha} = \Lambda^{-1} Z'y \tag{4}$$

And the mean squared error matrix (MSEM) of $\hat{\alpha}$is given by

$$MSEM\,(\hat{\alpha}) = \sigma^2\Lambda^{-1} \tag{5}$$

The ORR estimator α by Hoerl and Kennard [7] is defined as:

$$\hat{\alpha}_k = MX'y \tag{6}$$

where $M = (\Lambda + kI)^{-1}$ and k is a biasing parameter such that k > 0.

$$MSEM\,(\hat{\alpha}_k) = \sigma^2 M\Lambda M' + (M\Lambda - I)\,\alpha\alpha'\,(M\Lambda - I)' \tag{7}$$

The Liu estimator by Liu [8] is defined as:

$$\hat{\alpha}_d = S\,\hat{\alpha} \tag{8}$$

where $S = (\Lambda + I)^{-1} (\Lambda + dI)$ and d is a biasing parameter of the Liu Estimator.

$$MSEM\,(\hat{\alpha}_d) = \sigma^2 S\,\Lambda^{-1} S' + (S - I)\,\alpha\alpha'\,(S - I)' \tag{9}$$

The Modified One-Parameter Liu estimator by Lukman *et al.* [18]. is defined as:

$$\hat{\alpha}_{ML} = E\hat{\alpha} \tag{10}$$

where $E = (\Lambda + I)^{-1} (\Lambda - dI)$ and d is a biasing parameter of the Liu Estimator.

$$MSEM\,(\hat{\alpha}_{ML}) = \sigma^2 E\Lambda^{-1} E' + (E - I)\,\alpha\alpha'\,(E - I)' \tag{11}$$

The Kibria-Lukman (KL) estimator [11] is defined as:

$$\hat{\alpha}_{KL} = F\hat{\alpha} \tag{12}$$

where $F = (\Lambda + kI)^{-1}(\Lambda - kI)$ and

$$MSEM(\hat{\alpha}_{KL}) = \sigma^2 F\Lambda^{-1}F' + (F - I)\alpha\alpha'(F - I)' \tag{13}$$

The Dawoud-Kibria (DK) estimator [12] is defined as:

$$\hat{\alpha}_{DK} = H\hat{\alpha} \tag{14}$$

where $H = (\Lambda + k(1 + d)I)^{-1}(\Lambda - k(1 + d)I)$ and

$$MSEM(\hat{\alpha}_{DK}) = \sigma^2 H\Lambda^{-1}H' + (H - I)\alpha\alpha'(H - I)' \tag{15}$$

The proposed estimator

The Modified One-Parameter Liu [18] estimator $\alpha$is defined as:

$$\hat{\alpha}_{ML} = E\hat{\alpha}$$

Following the similar method proposed by Liu [19] and Kaciranlar *et al.* [20], the proposed Biased Two-Parameter estimator α_{BTP} is defined as:

$$\hat{\alpha}_{BTP} = EH\hat{\alpha} \tag{16}$$

Properties of the new estimator

The bias, covariance, and mean squared error matrix (MSEM) of the proposed estimator are given as follows:

$$B(\hat{\alpha}_{BTP}) = (EH - I)\alpha \tag{17}$$

$$D(\hat{\alpha}_{BTP}) = \sigma^2 EH\Lambda^{-1}E'H' \tag{18}$$

$$MSEM(\hat{\alpha}_{BTP}) = \sigma^2 EH\Lambda^{-1}E'H' + (EH - I)\alpha\alpha'(EH - I)' \tag{19}$$

The following lemmas are useful for making some theoretical comparisons and proving the statistical properties $\hat{\alpha}_{BTP}$.

Lemma 1: Let J be an n x n positive definite matrix, that is, J > 0, and αbe some vector, then $J - \alpha\alpha' \geq 0$ if and only if $\alpha' J^{-1}\alpha \leq 1$. Farebrother [21]

Lemma 2: Let $\hat{\alpha}_i = A_i y$ i = 1, 2 be two linear estimators of α. Suppose that $D = cov(\hat{\alpha}_1) - cov(\alpha_2) > 0$, where $cov(\hat{\alpha}_i)$ i = 1, 2 denotes the covariance matrix of $\hat{\alpha}_i$ and $b_i = Bias(\hat{\alpha}_i) = (A_i X - I)\alpha$, i =1, 2. Consequently,

$$\Delta(\hat{\alpha}_1 - \hat{\alpha}_2) = MSEM(\hat{\alpha}_1) - MSEM(\hat{\alpha}_2) = \sigma^2 D + b_1 b_1' - b_2 b_2' > 0 \tag{20}$$

If and only if $b_2'\left[\sigma^2 D + b_1 b_1'\right]^{-1} b_2 < 1$, where MSEM($\hat{\alpha}_i$) = $cov(\hat{\alpha}_i) + b_i b_i'$ Trenkler and Toutenburg [22]

The article was organized as follows. In section 2, a theoretical comparison of the proposed estimator with the existing ones is done. In section 3, biasing parameters are derived. A simulation study is conducted in Section 4 to evaluate the performance of the proposed estimator. A numerical example is given in Section 5 to illustrate the findings in the paper, and Section 6 ends with some concluding remarks.

2. Comparison among the Estimators

In this section, theoretical comparisons among the proposed BTP estimator and the OLS, ORR, Liu, KL, ML, and DK estimators

Comparison between $\hat{\alpha}$ and $\hat{\alpha}_{BTP}$.

The difference between $MSEM(\hat{\alpha}) - MSEM(\hat{\alpha}_{BTP})$ is given as follows:

$$MSEM(\hat{\alpha}) - MSEM(\hat{\alpha}_{MLKL}) = \sigma^2\Lambda^{-1} - \left(\sigma^2 EH\Lambda^{-1}E'H' + (EH - I)\alpha\alpha'(EH - I)'\right) \tag{21}$$

Let $k > 0$ and $0 < d < 1$. Thus, the following theorem holds.

Theorem 1: The estimator $\hat{\alpha}_{BTP}$is superior to the estimator $\hat{\alpha}$ if and only if

$$\alpha' (EH - I)' \left\lfloor \sigma^2 \left(\Lambda^{-1} - EH\Lambda^{-1}E'H'\right)\right\rfloor \alpha (EH - I) < 1 \tag{22}$$

Proof

$$D[\hat{\alpha}] - D(\hat{\alpha}_{BTP}) = \sigma^2 \left(\Lambda^{-1} - EH\Lambda^{-1}E'H'\right)$$

$$= \sigma^2 diag \left\{ \frac{1}{\lambda_i} - \frac{(\lambda_i - d)^2 (\lambda_i - k(I + d))^2}{\lambda_i (\lambda_i + 1)^2 (\lambda_i + k(1 + d))^2} \right\}_{i=1}^{p} \tag{23}$$

$\Lambda^{-1} - EH\Lambda^{-1}E'H'$ will be pdf if and only if $(\lambda_i + 1)^2 (\lambda_i + k(1 + d)^2 - (\lambda_i - d)^2 (\lambda_i - k(1 + d))^2 > 0$.

Comparison between $\hat{\alpha}_k$ and $\hat{\alpha}_{BTP}$.

The difference between $MSEM(\hat{\alpha}_k) - MSEM(\hat{\alpha}_{BTP})$ is given as follows:

$$MSEM(\hat{\alpha}_k) - MSEM(\hat{\alpha}_{BTP})$$

$$= \left(\sigma^2 M\Lambda M' + (M\Lambda - I)\alpha\alpha' (M\Lambda - I)'\right) - \left(\sigma^2 EH\Lambda^{-1}E'H' + (EH - I)\alpha\alpha' (EH - I)'\right) \tag{24}$$

Let $k > 0$ and $0 < d < 1$. Thus, the following theorem holds.

Theorem 3.2: The estimator $\hat{\alpha}_{BTP}$is superior to the estimator $\hat{\alpha}_k$using the MSEM criterion, that is, MSEM $(\hat{\alpha}_k)$ − MSEM $(\hat{\alpha}_{BTP}) > 0$ if and only if

$$\alpha' (EH - I)' \left\lfloor \sigma^2 \left(M\Lambda M' - EH\Lambda^{-1}E'H'\right) + (M\Lambda - I)\alpha\alpha' (M\Lambda - I)' \right\rfloor \alpha (EH - I) < 1 \tag{25}$$

Proof

$$D(\hat{\alpha}_k) - D(\hat{\alpha}_{BTP}) = \sigma^2 \left(M\Lambda M' - EH\Lambda^{-1}E'H'\right)$$

$$= \sigma^2 diag \left\{ \frac{\lambda_i}{(\lambda_i + k)^2} - \frac{(\lambda_i - d)^2 (\lambda_i - k(1 + d))^2}{\lambda_i (\lambda_i + 1)^2 (\lambda_i + k(1 + d))^2} \right\}_{i=1}^{p} \tag{26}$$

$M\Lambda M - EH\Lambda^{-1}E'H'$ will be pdf if and only if $\lambda_i^2 (\lambda_i + 1)^2 (\lambda_i + k(1 + d))^2 - (\lambda_i - d)^2 (\lambda_i - k)^2 (\lambda_i + k)^2 > 0$.

Comparison between $\hat{\alpha}_d$ and $\hat{\alpha}_{BTP}$.

The difference between $MSEM(\hat{\alpha}_d) - MSEM(\hat{\alpha}_{BTP})$ is given as follows:

$$MSEM(\hat{\alpha}_d) - MSEM(\hat{\alpha}_{BTP})$$

$$= \left(\sigma^2 S\Lambda^{-1}S' + (S - I)\alpha\alpha' (S - I)'\right) - \left(\sigma^2 EH\Lambda^{-1}E'H' + (EH - I)\alpha\alpha' (EH - I)'\right) \tag{27}$$

Let $k > 0$ and $0 < d < 1$. Thus, the following theorem holds.

Theorem 3.3: The estimator $\hat{\alpha}_{BTP}$is superior to the estimator $\hat{\alpha}_d$using the MSEM criterion, that is, $MSEM(\hat{\alpha}_d) - MSEM(\hat{\alpha}_{BTP}) > 0$if and only if

$$\alpha' (EH - I)' \left\lfloor \sigma^2 \left(S\Lambda^{-1}S' - EH\Lambda^{-1}E'H'\right) + (S - I)\alpha\alpha' (S - I)' \right\rfloor \alpha (EH - I) < 1 \tag{28}$$

Proof

$$D[\hat{\alpha}_d] - D(\hat{\alpha}_{BTP}) = \sigma^2 \left(S\Lambda^{-1}S' - EH\Lambda^{-1}E'H'\right)$$

$$= \sigma^2 diag \left\{ \frac{(\lambda_i + d)^2}{\lambda_i (\lambda_i + 1)^2} - \frac{(\lambda_i - d)^2 (\lambda_i - k(1 + d))^2}{\lambda_i (\lambda_i + 1)^2 (\lambda_i + k(1 + d))^2} \right\}_{i=1}^{p} \tag{29}$$

$S\Lambda^{-1}S' - EH\Lambda^{-1}E'H'$ will be pdf if and only if $(\lambda_i + d)^2 (\lambda_i + k(1 + d))^2 - (\lambda_i - d)^2 (\lambda_i - k(1 + d))^2 > 0$.

Comparison between $\hat{\alpha}_{ML}$ and $\hat{\alpha}_{BTP}$.

The difference between $MSEM(\hat{\alpha}_{ML}) - MSEM(\hat{\alpha}_{BTP})$ is given as follows:

$$MSEM(\hat{\alpha}_{ML}) - MSEM(\hat{\alpha}_{BTP}) = \left(\sigma^2 E\Lambda^{-1}E' + (E - I)\alpha\alpha' (E - I)'\right) - \left(\sigma^2 EH\Lambda^{-1}E'H' + (EH - I)\alpha\alpha' (EH - I)'\right) (30) \tag{30}$$

Let $k > 0$ and $0 < d < 1$. Thus, the following theorem holds.

Theorem 3.4: The estimator $\hat{\alpha}_{BTP}$is superior to the estimator $\hat{\alpha}_{ML}$using the MSEM criterion, that is, MSEM ($\hat{\alpha}_{ML}$) − MSEM ($\hat{\alpha}_{BTP}$) > 0 if and only if

$$\alpha' (EH - I)' \left\lfloor \sigma^2 \left(E\Lambda^{-1}E' - EH\Lambda^{-1}E'H' \right) + (E - I)\, \alpha\alpha' \,(E - I)' \right\rfloor \alpha \,(EH - I) < 1 \tag{31}$$

Proof

$$\begin{aligned} D\left[\hat{\alpha}_{ML}\right] - D\left(\hat{\alpha}_{BTP}\right) &= \sigma^2 \left(E\Lambda^{-1}E' - EH\Lambda^{-1}E'H' \right) \\ &= \sigma^2 diag \left\{ \frac{(\lambda_i - d)^2}{\lambda_i (\lambda_i + 1)^2} - \frac{(\lambda_i - d)^2 (\lambda_i - k(1+d))^2}{\lambda_i (\lambda_i + 1)^2 (\lambda_i + k(1+d))^2} \right\}_{i=1}^{p} \end{aligned} \tag{32}$$

$E\Lambda^{-1}E' - EH\Lambda^{-1}E'H'$ will be pdf if and only if $(\lambda_i - d)^2 (\lambda_i + k(1+d))^2 - (\lambda_i - d)^2 (\lambda_i - k(1+d))^2 > 0$.

Comparison between $\hat{\alpha}_{KL}$ and $\hat{\alpha}_{BTP}$.

The difference between $MSEM(\hat{\alpha}_{KL}) - MSEM(\hat{\alpha}_{BTP})$ is given as follows:

$$MSEM(\hat{\alpha}_{KL}) - MSEM(\hat{\alpha}_{BTP}) = \left(\sigma^2 F\Lambda^{-1}F' + (F - I)\, \alpha\alpha' \,(F - I)' \right) - \left(\sigma^2 EH\Lambda^{-1}E'H' + (EH - I)\, \alpha\alpha' \,(EH - I)' \right) \tag{33}$$

Let $k > 0$ and $0 < d < 1$. Thus, the following theorem holds.

Theorem 3.5: The estimator $\hat{\alpha}_{BTP}$is superior to the estimator $\hat{\alpha}_{KL}$using the MSEM criterion, that is, MSEM ($\hat{\alpha}_{KL}$) − MSEM ($\hat{\alpha}_{BTP}$) > 0 if and only if

$$\alpha' (EH - I)' \left\lfloor \sigma^2 \left(F\Lambda^{-1}F' - EH\Lambda^{-1}E'H' \right) + (F - I)\, \alpha\alpha' \,(F - I)' \right\rfloor \alpha \,(EH - I) < 1 \tag{34}$$

Proof

$$\begin{aligned} D\left[\hat{\alpha}_{KL}\right] - D\left(\hat{\alpha}_{BTP}\right) &= \sigma^2 \left(F\Lambda^{-1}F' - EH\Lambda^{-1}E'H' \right) \\ &= \sigma^2 diag \left\{ \frac{(\lambda_i - k)^2}{\lambda_i (\lambda_i + k)^2} - \frac{(\lambda_i - d)^2 (\lambda_i - k(1+d))^2}{\lambda_i (\lambda_i + 1)^2 (\lambda_i + k(1+d))^2} \right\}_{i=1}^{p} \end{aligned} \tag{35}$$

$F\Lambda^{-1}F' - EH\Lambda^{-1}E'H'$ will be pdf if and only if $(\lambda_i - k)^2 (\lambda_i + 1)^2 (\lambda_i + k(1+d))^2 - (\lambda_i - d)^2 (\lambda_i - k(1+d))^2 (\lambda_i + k)^2 > 0$.

Comparison between $\hat{\alpha}_{DK}$ and $\hat{\alpha}_{BTP}$.

The difference between $MSEM(\hat{\alpha}_{DK}) - MSEM(\hat{\alpha}_{BTP})$ is given as follows:

$$MSEM(\hat{\alpha}_{DK}) - MSEM(\hat{\alpha}_{BTP}) = \left(\sigma^2 H\Lambda^{-1}H' + (H - I)\, \alpha\alpha' \,(H - I)' \right) - \left(\sigma^2 EH\Lambda^{-1}E'H' + (EH - I)\, \alpha\alpha' \,(EH - I)' \right) \tag{36}$$

Let $k > 0$ and $0 < d < 1$. Thus, the following theorem holds.

Theorem 3.6: The estimator $\hat{\alpha}_{BTP}$is superior to the estimator $\hat{\alpha}_{DK}$using the MSEM criterion, that is, MSEM ($\hat{\alpha}_{DK}$) − MSEM ($\hat{\alpha}_{BTP}$) > 0 if and only if

$$\alpha' (EH - I)' \left\lfloor \sigma^2 \left(H\Lambda^{-1}H' - EH\Lambda^{-1}E'H' \right) + (H - I)\, \alpha\alpha' \,(H - I)' \right\rfloor \alpha \,(EH - I) < 1 \tag{37}$$

Proof

$$\begin{aligned} D\left[\hat{\alpha}_{DK}\right] - D\left(\hat{\alpha}_{BTP}\right) &= \sigma^2 \left(H\Lambda^{-1}H' - EH\Lambda^{-1}E'H' \right) \\ &= \sigma^2 diag \left\{ \frac{(\lambda_i - k(1+d))^2}{\lambda_i (\lambda_i + k(1+d))^2} - \frac{(\lambda_i - d)^2 (\lambda_i - k(1+d))^2}{\lambda_i (\lambda_i + 1)^2 (\lambda_i + k(1+d))^2} \right\}_{i=1}^{p} \end{aligned} \tag{38}$$

$H\Lambda^{-1}H' - EH\Lambda^{-1}E'H'$ will be pdf if and only if $(\lambda_i - k(1+d))^2 (\lambda_i + 1)^2 - (\lambda_i - d)^2 (\lambda_i - k(1+d))^2 > 0$.

3. Selection of Biasing Parameters

For practical use, there is a need to estimate the parameters k and d. Different researchers have proposed different estimators of k and d for different types of regression models. Following some of these researchers like Hoerl *et al.* [7], Liu [8], Lukman and Ayinde [23], Ayinde *et al.* [24], Owolabi *et al.* [25]. and Dawoud *et al*. [12, 26] to mention a few, among others. In determining the value of k, d is fixed such that k is obtained as follows:

$$MSEM(\hat{\alpha}_{BTP}) = \sigma^2 EH\Lambda^{-1}E'H' + (EH - I)\alpha\alpha'(EH - I)'$$

$$l(k,d) = MSEM(\hat{\alpha}_{BTP}) = tr[MSEM(\hat{\alpha}_{BTP})]$$

$$l(k,d) = \sigma^2 \sum_{i}^{p} \frac{(\lambda_i - d)^2 (\lambda_i - k(1+d))^2}{\lambda_i (\lambda_i + 1)^2 (\lambda_i + k(1+d))^2} + \sum_{i}^{p} \frac{(2\lambda_i k + \lambda_i d - kd + \lambda_i + k(1+d))^2 \alpha_i^2}{(\lambda_i + 1)^2 (\lambda_i + k(1+d))^2} \tag{39}$$

Differentiating $l(k,d)$with respect to k and setting $\frac{\partial l(k,d)}{\partial k} = 0$, we have

$$k = \frac{\sigma^2 \lambda_i (d - \lambda_i) + \alpha_i^2 \lambda_i^2 (d+1)}{\sigma^2 (1+d)(d - \lambda_i) - \alpha_i^2 \lambda_i (1+d)(2\lambda_i - d + 1)} \tag{40}$$

Also, the value of d is found by differentiating $l(k,d)$with respect to d; for a fixed k and setting $\frac{\partial l(k,d)}{\partial d} = 0$, we obtain

$$d = \frac{-\left(\sigma^2\lambda_i - \sigma^2 k + \alpha_i^2\lambda_i^2 + \sigma^2\lambda_i k + 2\alpha_i^2\lambda_i^2 k\right)}{2\left(\sigma^2 k + \alpha_i^2\lambda_i k\right)} + \frac{\sqrt{(\alpha_i^2)^2\lambda_i^2 k(2\lambda_i k + k + \lambda_i) + \sigma^2\alpha_i^2\lambda_i^2 k(k - \lambda_i) + \sigma^2\alpha_i^2\lambda_i k(2\sigma^2\lambda_i k + k + \lambda_i) + (\sigma^2)^2\lambda_i k(k - \lambda_i)}}{\left(\sigma^2 k + \alpha_i^2\lambda_i k\right)} \tag{41}$$

For the practical purpose $\sigma$and $\alpha$are replaced with $\hat{\sigma}$and $\hat{\alpha}$respectively in (40) and (41)

$$\hat{k} = \frac{\hat{s}^2 \lambda_i (d - \lambda_i) + \hat{a}_i^2 \lambda_i^2 (d+1)}{\hat{s}^2 (1+d)(d - \lambda_i) - \hat{a}_i^2 \lambda_i (1+d)(2\lambda_i - d + 1)} \tag{42}$$

$$\hat{d} = \frac{-\left(\hat{\sigma}^2\lambda_i - \hat{\sigma}^2 k + \hat{\alpha}_i^2\lambda_i^2 + \hat{\sigma}^2\lambda_i k + 2\hat{\alpha}_i^2\lambda_i^2 k\right)}{2\left(\hat{\sigma}^2 k + \hat{\alpha}_i^2\lambda_i k\right)} + \frac{\sqrt{(\hat{\alpha}_i^2)^2\lambda_i^2 k(2\lambda_i k + k + \lambda_i) + \hat{\sigma}^2\hat{\alpha}_i^2\lambda_i^2 k(k - \lambda_i) + \hat{\sigma}^2\hat{\alpha}_i^2\lambda_i k(2\hat{\sigma}^2\lambda_i k + k + \lambda_i) + (\hat{\sigma}^2)^2\lambda_i k(k - \lambda_i)}}{\left(\hat{\sigma}^2 k + \hat{\alpha}_i^2\lambda_i k\right)} \tag{43}$$

4. Simulation Techniques

To study the performance of the estimators, a simulation has been carried out in this section. Following McDonald and Galarneau [27], Gibbons [28], and Lukman *et al.* [29], the explanatory variables are generated using the equation:

$$x_{ij} = \left(1 - \rho^2\right)^{1/2} z_{ij} + \rho z_{i,p+1}, i = 1,2,...,n, \quad j = 1,2,...,p. \tag{44}$$

where z_{ij} are independent standard normal pseudo-random numbers, ρ represents the correlation between two explanatory variables, and p is the number of explanatory variables. In this study, ρ is considered to be 0.80, 0.90, 0.95, and 0.99. We consider p = 3 and 7 in the simulation. The following equation determines the n observations for the dependent variable y:

$$Y_i = \beta_1 X_1 + \beta_2 X_2 + ... + \beta_p X_p + e_i \tag{45}$$

where e_i is normally distributed with mean 0 and variance σ^2 Firinguetti [30]. According to Newhouse and Oman, we choose βsuch that $\beta'\beta$=1 [31]. The chosen values of $\sigma$are 3, 5, and 10, and the replication for the study is 1000 times. The mean square error is then obtained as:

$$MSE(\hat{\alpha}) = \frac{1}{1000} \sum_{j=1}^{1000} \left(\hat{\alpha}_{ij} - \alpha_i\right)' \left(\hat{\alpha}_{ij} - \alpha_i\right) \tag{46}$$

Table 1: Estimated MSE when n=50 and p=3

K	d	sigma	rho	OLS	RIDGE	LIU	K-L	DK	ML	BTP
0.3	0.2	3	0.8	1.227	1.1802	1.1098	1.1344	1.1168	1.0535	**0.9598**
			0.9	2.2158	2.0587	1.8412	1.9076	1.8514	1.6674	**1.3965**
			0.95	4.2136	3.6579	2.9999	3.1428	2.964	2.4725	**1.7516**
			0.99	20.175	10.956	6.5009	4.5913	3.3199	2.5317	**0.4895**
		5	0.8	3.4082	3.2782	3.0825	3.1509	3.1019	2.9261	**2.6655**
			0.9	6.1551	5.7185	5.1142	5.2987	5.1426	4.6314	**3.8787**
			0.95	11.704	10.161	8.333	8.73	8.2333	6.868	**4.8656**
			0.99	56.042	30.434	18.058	12.754	9.2219	7.0326	**1.3598**
		10	0.8	13.633	13.113	12.329	12.603	12.407	11.704	**10.661**
			0.9	24.62	22.874	20.457	21.194	20.57	18.525	**15.513**
			0.95	46.818	40.644	33.332	34.92	32.933	27.472	**19.463**
			0.99	224.17	121.73	72.233	51.015	36.888	28.131	**5.4392**
	0.5	3	0.8	1.227	1.1802	1.153	1.1344	1.0909	1.0124	**0.9015**
			0.9	2.2158	2.0587	1.9774	1.9076	1.7703	1.543	**1.2377**
			0.95	4.2136	3.6579	3.4302	3.1428	2.7145	2.1117	**1.3776**
			0.99	20.175	10.956	10.733	4.5913	1.9692	0.8096	**0.1418**
		5	0.8	3.4082	3.2782	3.2026	3.1509	3.03	2.8117	**2.5032**
			0.9	6.1551	5.7185	5.4928	5.2987	4.9173	4.2857	**3.4373**
			0.95	11.704	10.161	9.5284	8.73	7.5403	5.8658	**3.8266**
			0.99	56.042	30.434	29.813	12.754	5.4701	2.2488	**0.3937**
		10	0.8	13.633	13.113	12.81	12.603	12.119	11.246	**10.011**
			0.9	24.62	22.874	21.971	21.194	19.669	17.142	**13.748**
			0.95	46.818	40.644	38.114	34.92	30.161	23.464	**15.306**
			0.99	224.17	121.73	119.25	51.015	21.881	8.9955	**1.5749**
	0.8	3	0.8	1.227	1.1802	1.1971	1.1344	1.0656	0.9721	**0.8463**
			0.9	2.2158	2.0587	2.1188	1.9076	1.6928	1.4237	**1.0944**
			0.95	4.2136	3.6579	3.8903	3.1428	2.4857	1.7807	**1.0724**
			0.99	20.175	10.956	16.04	4.5913	1.1027	0.1629	**0.0567**
		5	0.8	3.4082	3.2782	3.3251	3.1509	2.9597	2.6997	**2.3495**
			0.9	6.1551	5.7185	5.8855	5.2987	4.702	3.9542	**3.0389**
			0.95	11.704	10.161	10.806	8.73	6.9046	4.9464	**2.9786**
			0.99	56.042	30.434	44.554	12.754	3.063	0.4524	**0.1572**
		10	0.8	13.633	13.113	13.3	12.603	11.838	10.797	**9.3953**
			0.9	24.62	22.874	23.542	21.194	18.808	15.816	**12.154**
			0.95	46.818	40.644	43.226	34.92	27.619	19.786	**11.914**
			0.99	224.17	121.73	178.22	51.015	12.252	1.8099	**0.6288**
0.6	0.2	3	0.8	1.227	1.1362	1.1098	1.0491	1.0169	1.0535	**0.8749**
			0.9	2.2158	1.9183	1.8412	1.6431	1.548	1.6674	**1.1708**
			0.95	4.2136	3.208	2.9999	2.3436	2.0827	2.4725	**1.2415**
			0.99	20.175	6.9114	6.5009	0.721	0.2939	2.5317	**0.0876**
		5	0.8	3.4082	3.1559	3.0825	2.9139	2.8244	2.9261	**2.4293**
			0.9	6.1551	5.3284	5.1142	4.5638	4.2996	4.6314	**3.2513**
			0.95	11.704	8.9111	8.333	6.5101	5.7852	6.868	**3.4484**
			0.99	56.042	19.198	18.058	2.0029	0.8163	7.0326	**0.2433**
		10	0.8	13.633	12.623	12.329	11.655	11.296	11.704	**9.7148**
			0.9	24.62	21.313	20.457	18.255	17.198	18.525	**13.003**
			0.95	46.818	35.645	33.332	26.041	23.141	27.472	**13.794**
			0.99	224.17	76.794	72.233	8.0117	3.2652	28.131	**0.973**
	0.5	3	0.8	1.227	1.1362	1.153	1.0491	0.9706	1.0124	**0.8035**

			0.9	2.2158	1.9183	1.9774	1.6431	1.4156	1.543	**0.9943**
			0.95	4.2136	3.208	3.4302	2.3436	1.7424	2.1117	**0.8988**
			0.99	20.175	6.9114	10.733	0.721	0.2033	0.8096	**0.0512**
		5	0.8	3.4082	3.1559	3.2026	2.9139	2.6954	2.8117	**2.2303**
			0.9	6.1551	5.3284	5.4928	4.5638	3.9317	4.2857	**2.7607**
			0.95	11.704	8.9111	9.5284	6.5101	4.84	5.8658	**2.4962**
			0.99	56.042	19.198	29.813	2.0029	0.5647	2.2488	**0.1419**
		10	0.8	13.633	12.623	12.81	11.655	10.78	11.246	**8.9179**
			0.9	24.62	21.313	21.971	18.255	15.726	17.142	**11.04**
			0.95	46.818	35.645	38.114	26.041	19.36	23.464	**9.9845**
			0.99	224.17	76.794	119.25	8.0117	2.2591	8.9955	**0.5672**
	0.8	3	0.8	1.227	1.1362	1.1971	1.0491	0.9264	0.9721	**0.7377**
			0.9	2.2158	1.9183	2.1188	1.6431	1.2944	1.4237	**0.843**
			0.95	4.2136	3.208	3.8903	2.3436	1.4547	1.7807	**0.6458**
			0.99	20.175	6.9114	16.04	0.721	0.4881	0.1629	**0.0473**
		5	0.8	3.4082	3.1559	3.3251	2.9139	2.5725	2.6997	**2.047**
			0.9	6.1551	5.3284	5.8855	4.5638	3.595	3.9542	**2.3401**
			0.95	11.704	8.9111	10.806	6.5101	4.0406	4.9464	**1.7932**
			0.99	56.042	19.198	44.554	2.0029	1.3556	0.4524	**0.1306**
		10	0.8	13.633	12.623	13.3	11.655	10.288	10.797	**8.1834**
			0.9	24.62	21.313	23.542	18.255	14.379	15.816	**9.3569**
			0.95	46.818	35.645	43.226	26.041	16.163	19.786	**7.1724**
			0.99	224.17	76.794	178.22	8.0117	5.4227	1.8099	**0.522**
0.9	0.2	3	0.8	1.227	1.0947	1.1098	0.9706	0.9264	1.0535	**0.798**
			0.9	2.2158	1.7923	1.8412	1.4156	1.2944	1.6674	**0.982**
			0.95	4.2136	2.8383	2.9999	1.7424	1.4547	2.4725	**0.8767**
			0.99	20.175	4.7705	6.5009	0.2033	0.4881	2.5317	**0.089**
		5	0.8	3.4082	3.0407	3.0825	2.6954	2.5725	2.9261	**2.2149**
			0.9	6.1551	4.9784	5.1142	3.9317	3.595	4.6314	**2.7266**
			0.95	11.704	7.8841	8.333	4.84	4.0406	6.868	**2.4349**
			0.99	56.042	13.251	18.058	0.5647	1.3556	7.0326	**0.2469**
		10	0.8	13.633	12.162	12.329	10.78	10.288	11.704	**8.8563**
			0.9	24.62	19.913	20.457	15.726	14.379	18.525	**10.904**
			0.95	46.818	31.537	33.332	19.36	16.163	27.472	**9.7393**
			0.99	224.17	53.006	72.233	2.2591	5.4227	28.131	**0.9873**
	0.5	3	0.8	1.227	1.0947	1.153	0.9706	0.864	1.0124	**0.7167**
			0.9	2.2158	1.7923	1.9774	1.4156	1.1316	1.543	**0.7992**
			0.95	4.2136	2.8383	3.4302	1.7424	1.1038	2.1117	**0.5823**
			0.99	20.175	4.7705	10.733	0.2033	1.2476	0.8096	**0.0759**
		5	0.8	3.4082	3.0407	3.2026	2.6954	2.3989	2.8117	**1.9884**
			0.9	6.1551	4.9784	5.4928	3.9317	3.1425	4.2857	**2.2182**
			0.95	11.704	7.8841	9.5284	4.84	3.066	5.8658	**1.6168**
			0.99	56.042	13.251	29.813	0.5647	3.4652	2.2488	**0.2102**
		10	0.8	13.633	12.162	12.81	10.78	9.5932	11.246	**7.9487**
			0.9	24.62	19.913	21.971	15.726	12.568	17.142	**8.8689**
			0.95	46.818	31.537	38.114	19.36	12.264	23.464	**6.4666**
			0.99	224.17	53.006	119.25	2.2591	13.861	8.9955	**0.84**
	0.8	3	0.8	1.227	1.0947	1.1971	0.9706	0.806	0.9721	**0.6437**
			0.9	2.2158	1.7923	2.1188	1.4156	0.9888	1.4237	**0.6499**
			0.95	4.2136	2.8383	3.8903	1.7424	0.8309	1.7807	**0.3852**
			0.99	20.175	4.7705	16.04	0.2033	2.1589	0.1629	**0.0537**
		5	0.8	3.4082	3.0407	3.3251	2.6954	2.2373	2.6997	**1.785**

			0.9	6.1551	4.9784	5.8855	3.9317	2.7455	3.9542	**1.8027**
			0.95	11.704	7.8841	10.806	4.84	2.3076	4.9464	**1.0688**
			0.99	56.042	13.251	44.554	0.5647	5.9966	0.4524	**0.148**
		10	0.8	13.633	12.162	13.3	10.78	8.946	10.797	**7.1334**
			0.9	24.62	19.913	23.542	15.726	10.979	15.816	**7.2057**
			0.95	46.818	31.537	43.226	19.36	9.23	19.786	**4.2739**
			0.99	224.17	53.006	178.22	2.2591	23.986	1.8099	**0.5908**

NOTE: Minimum MSE value is bolded in each row

Table 2: Estimated MSE when n=100 and p=3

K	d	sigma	rho	OLS	RIDGE	LIU	K-L	DK	ML	BTP
0.3	0.2	3	0.8	0.5596	0.5499	0.5345	0.5403	0.5366	0.5223	**0.501**
			0.9	1.0178	0.9839	0.9322	0.9506	0.9378	0.8909	**0.8214**
			0.95	1.9464	1.8211	1.6447	1.7002	1.6549	1.5038	**1.2811**
			0.99	9.4002	6.9	4.8107	4.7956	4.1841	3.0985	**1.42**
		5	0.8	1.5543	1.5275	1.4848	1.5009	1.4905	1.4507	**1.3915**
			0.9	2.8273	2.7332	2.5896	2.6407	2.6049	2.4748	**2.2817**
			0.95	5.4066	5.0586	4.5687	4.7227	4.597	4.1773	**3.5585**
			0.99	26.112	19.167	13.363	13.321	11.623	8.607	**3.9442**
		10	0.8	6.2173	6.11	5.9393	6.0036	5.9618	5.8028	**5.5657**
			0.9	11.309	10.933	10.358	10.563	10.42	9.8992	**9.1265**
			0.95	21.626	20.235	18.275	18.891	18.388	16.709	**14.234**
			0.99	104.45	76.667	53.452	53.285	46.49	34.428	**15.777**
	0.5	3	0.8	0.5596	0.5499	0.5438	0.5403	0.531	0.5132	**0.4872**
			0.9	1.0178	0.9839	0.9639	0.9506	0.9188	0.8606	**0.7778**
			0.95	1.9464	1.8211	1.7548	1.7002	1.5894	1.4025	**1.1488**
			0.99	9.4002	6.9	6.3497	4.7956	3.4012	2.0693	**0.7952**
		5	0.8	1.5543	1.5275	1.5107	1.5009	1.4749	1.4254	**1.3532**
			0.9	2.8273	2.7332	2.6774	2.6407	2.5523	2.3906	**2.1603**
			0.95	5.4066	5.0586	4.8743	4.7227	4.4148	3.8957	**3.191**
			0.99	26.112	19.167	17.638	13.321	9.4477	5.7479	**2.2088**
		10	0.8	6.2173	6.11	6.0427	6.0036	5.8998	5.7016	**5.4125**
			0.9	11.309	10.933	10.71	10.563	10.209	9.5622	**8.6409**
			0.95	21.626	20.235	19.497	18.891	17.659	15.583	**12.764**
			0.99	104.45	76.667	70.552	53.285	37.791	22.991	**8.8349**
	0.8	3	0.8	0.5596	0.5499	0.5532	0.5403	0.5255	0.5042	**0.4738**
			0.9	1.0178	0.9839	0.9961	0.9506	0.9003	0.8309	**0.7361**
			0.95	1.9464	1.8211	1.8685	1.7002	1.5264	1.3049	**1.0282**
			0.99	9.4002	6.9	8.1072	4.7956	2.7536	1.2584	**0.4166**
		5	0.8	1.5543	1.5275	1.5368	1.5009	1.4596	1.4004	**1.3158**
			0.9	2.8273	2.7332	2.7668	2.6407	2.5007	2.3079	**2.0445**
			0.95	5.4066	5.0586	5.1902	4.7227	4.24	3.6245	**2.8558**
			0.99	26.112	19.167	22.52	13.321	7.649	3.4955	**1.157**
		10	0.8	6.2173	6.11	6.1472	6.0036	5.8384	5.6014	**5.263**
			0.9	11.309	10.933	11.067	10.563	10.003	9.2314	**8.1776**
			0.95	21.626	20.235	20.761	18.891	16.96	14.498	**11.423**
			0.99	104.45	76.667	90.08	53.285	30.596	13.982	**4.6276**
0.6	0.2	3	0.8	0.5596	0.5405	0.5345	0.5218	0.5146	0.5223	**0.4806**
			0.9	1.0178	0.9518	0.9322	0.8881	0.8643	0.8909	**0.7576**
			0.95	1.9464	1.708	1.6447	1.4859	1.408	1.5038	**1.0922**

			0.99	9.4002	5.2929	4.8107	2.3853	1.775	3.0985	**0.63**
		5	0.8	1.5543	1.5014	1.4848	1.4495	1.4294	1.4507	**1.3348**
			0.9	2.8273	2.6439	2.5896	2.4669	2.4008	2.4748	**2.1043**
			0.95	5.4066	4.7444	4.5687	4.1273	3.911	4.1773	**3.0336**
			0.99	26.112	14.702	13.363	6.6258	4.9303	8.607	**1.7499**
		10	0.8	6.2173	6.0056	5.9393	5.7978	5.7176	5.8028	**5.3391**
			0.9	11.309	10.575	10.358	9.8677	9.6032	9.8992	**8.417**
			0.95	21.626	18.978	18.275	16.509	15.644	16.709	**12.134**
			0.99	104.45	58.81	53.452	26.503	19.721	34.428	**6.9991**
	0.5	3	0.8	0.5596	0.5405	0.5438	0.5218	0.504	0.5132	**0.4627**
			0.9	1.0178	0.9518	0.9639	0.8881	0.8299	0.8606	**0.7033**
			0.95	1.9464	1.708	1.7548	1.4859	1.2988	1.4025	**0.9421**
			0.99	9.4002	5.2929	6.3497	2.3853	1.1091	2.0693	**0.2901**
		5	0.8	1.5543	1.5014	1.5107	1.4495	1.3999	1.4254	**1.2849**
			0.9	2.8273	2.6439	2.6774	2.4669	2.3051	2.3906	**1.9533**
			0.95	5.4066	4.7444	4.8743	4.1273	3.6077	3.8957	**2.6165**
			0.99	26.112	14.702	17.638	6.6258	3.0808	5.7479	**0.8054**
		10	0.8	6.2173	6.0056	6.0427	5.7978	5.5996	5.7016	**5.1392**
			0.9	11.309	10.575	10.71	9.8677	9.2202	9.5622	**7.8126**
			0.95	21.626	18.978	19.497	16.509	14.431	15.583	**10.465**
			0.99	104.45	58.81	70.552	26.503	12.323	22.991	**3.2209**
	0.8	3	0.8	0.5596	0.5405	0.5532	0.5218	0.4936	0.5042	**0.4455**
			0.9	1.0178	0.9518	0.9961	0.8881	0.7968	0.8309	**0.6527**
			0.95	1.9464	1.708	1.8685	1.4859	1.1981	1.3049	**0.8114**
			0.99	9.4002	5.2929	8.1072	2.3853	0.6642	1.2584	**0.1322**
		5	0.8	1.5543	1.5014	1.5368	1.4495	1.3711	1.4004	**1.2368**
			0.9	2.8273	2.6439	2.7668	2.4669	2.2133	2.3079	**1.8126**
			0.95	5.4066	4.7444	5.1902	4.1273	3.3278	3.6245	**2.2533**
			0.99	26.112	14.702	22.52	6.6258	1.845	3.4955	**0.3668**
		10	0.8	6.2173	6.0056	6.1472	5.7978	5.4841	5.6014	**4.9466**
			0.9	11.309	10.575	11.067	9.8677	8.853	9.2314	**7.2496**
			0.95	21.626	18.978	20.761	16.509	13.311	14.498	**9.0125**
			0.99	104.45	58.81	90.08	26.503	7.3796	13.982	**1.4662**
0.9	0.2	3	0.8	0.5596	0.5314	0.5345	0.504	0.4936	0.5223	**0.4612**
			0.9	1.0178	0.9213	0.9322	0.8299	0.7968	0.8909	**0.699**
			0.95	1.9464	1.6055	1.6447	1.2988	1.1981	1.5038	**0.9314**
			0.99	9.4002	4.1956	4.8107	1.1091	0.6642	3.0985	**0.2576**
		5	0.8	1.5543	1.476	1.4848	1.3999	1.3711	1.4507	**1.2807**
			0.9	2.8273	2.5591	2.5896	2.3051	2.2133	2.4748	**1.9414**
			0.95	5.4066	4.4597	4.5687	3.6077	3.3278	4.1773	**2.5869**
			0.99	26.112	11.654	13.363	3.0808	1.845	8.607	**0.7152**
		10	0.8	6.2173	5.9041	5.9393	5.5996	5.4841	5.8028	**5.1224**
			0.9	11.309	10.236	10.358	9.2202	8.853	9.8992	**7.765**
			0.95	21.626	17.839	18.275	14.431	13.311	16.709	**10.347**
			0.99	104.45	46.617	53.452	12.323	7.3796	34.428	**2.86**
	0.5	3	0.8	0.5596	0.5314	0.5438	0.504	0.4785	0.5132	**0.4396**
			0.9	1.0178	0.9213	0.9639	0.8299	0.7498	0.8606	**0.6363**
			0.95	1.9464	1.6055	1.7548	1.2988	1.0612	1.4025	**0.7727**
			0.99	9.4002	4.1956	6.3497	1.1091	0.2798	2.0693	**0.0972**
		5	0.8	1.5543	1.476	1.5107	1.3999	1.329	1.4254	**1.2204**
			0.9	2.8273	2.5591	2.6774	2.3051	2.0826	2.3906	**1.7669**
			0.95	5.4066	4.4597	4.8743	3.6077	2.9476	3.8957	**2.1459**

			0.99	26.112	11.654	17.638	3.0808	0.777	5.7479	**0.2695**
		10	0.8	6.2173	5.9041	6.0427	5.5996	5.3157	5.7016	**4.8807**
			0.9	11.309	10.236	10.71	9.2202	8.3302	9.5622	**7.0668**
			0.95	21.626	17.839	19.497	14.431	11.79	15.583	**8.5828**
			0.99	104.45	46.617	70.552	12.323	3.1074	22.991	**1.0773**
	0.8	3	0.8	0.5596	0.5314	0.5532	0.504	0.4639	0.5042	**0.419**
			0.9	1.0178	0.9213	0.9961	0.8299	0.7057	0.8309	**0.5792**
			0.95	1.9464	1.6055	1.8685	1.2988	0.9397	1.3049	**0.6404**
			0.99	9.4002	4.1956	8.1072	1.1091	0.1189	1.2584	**0.0471**
		5	0.8	1.5543	1.476	1.5368	1.3999	1.2883	1.4004	**1.163**
			0.9	2.8273	2.5591	2.7668	2.3051	1.9599	2.3079	**1.608**
			0.95	5.4066	4.4597	5.1902	3.6077	2.6099	3.6245	**1.7781**
			0.99	26.112	11.654	22.52	3.0808	0.3301	3.4955	**0.1302**
		10	0.8	6.2173	5.9041	6.1472	5.5996	5.1528	5.6014	**4.6506**
			0.9	11.309	10.236	11.067	9.2202	7.839	9.2314	**6.4308**
			0.95	21.626	17.839	20.761	14.431	10.439	14.498	**7.111**
			0.99	104.45	46.617	90.08	12.323	1.3197	13.982	**0.5194**

NOTE: Minimum MSE value is bolded in each row

Table 3: Estimated MSE when n=50 and p=7

K	d	sigma	rho	OLS	RIDGE	LIU	K-L	DK	ML	BTP
0.3	0.2	3	0.8	3.3789	3.2362	3.0254	3.097	3.0438	2.8569	**2.5803**
			0.9	6.3296	5.8369	5.1772	5.3669	5.1947	4.6497	**3.8479**
			0.95	12.278	10.524	8.5534	8.9216	8.3799	6.9701	**4.8929**
			0.99	59.959	31.635	19.093	13.216	9.8364	7.675	**1.9214**
		5	0.8	9.3859	8.9894	8.4038	8.6027	8.4551	7.9359	**7.1673**
			0.9	17.582	16.214	14.381	14.908	14.43	12.916	**10.689**
			0.95	34.106	29.232	23.759	24.782	23.278	19.361	**13.591**
			0.99	166.55	87.876	53.037	36.712	27.323	21.319	**5.3373**
		10	0.8	37.544	35.958	33.615	34.411	33.82	31.744	**28.669**
			0.9	70.329	64.855	57.524	59.633	57.72	51.664	**42.754**
			0.95	136.42	116.93	95.037	99.129	93.11	77.444	**54.364**
			0.99	666.21	351.5	212.15	146.85	109.29	85.278	**21.349**
	0.5	3	0.8	3.3789	3.2362	3.1553	3.097	2.9661	2.7343	**2.4101**
			0.9	6.3296	5.8369	5.5941	5.3669	4.9479	4.2755	**3.3877**
			0.95	12.278	10.524	9.8629	8.9216	7.633	5.9046	**3.8496**
			0.99	59.959	31.635	31.601	13.216	6.3565	3.055	**0.732**
		5	0.8	9.3859	8.9894	8.7648	8.6027	8.2391	7.5951	**6.6946**
			0.9	17.582	16.214	15.539	14.908	13.744	11.876	**9.4102**
			0.95	34.106	29.232	27.397	24.782	21.203	16.402	**10.693**
			0.99	166.55	87.876	87.781	36.712	17.657	8.486	**2.0333**
		10	0.8	37.544	35.958	35.059	34.411	32.956	30.38	**26.778**
			0.9	70.329	64.855	62.157	59.633	54.977	47.505	**37.641**
			0.95	136.42	116.93	109.59	99.129	84.811	65.606	**42.772**
			0.99	666.21	351.5	351.12	146.85	70.628	33.944	**8.1334**
	0.8	3	0.8	3.3789	3.2362	3.2884	3.097	2.8906	2.6147	**2.2505**
			0.9	6.3296	5.8369	6.0293	5.3669	4.7139	3.9195	**2.979**
			0.95	12.278	10.524	11.277	8.9216	6.9571	4.9438	**3.0171**
			0.99	59.959	31.635	47.489	13.216	4.2145	1.8153	**0.2557**
		5	0.8	9.3859	8.9894	9.1346	8.6027	8.0293	7.263	**6.251**

			0.9	17.582	16.214	16.748	14.908	13.094	10.887	**8.2749**
			0.95	34.106	29.232	31.325	24.782	19.325	13.733	**8.3805**
			0.99	166.55	87.876	131.91	36.712	11.707	5.0426	**0.7101**
		10	0.8	37.544	35.958	36.538	34.411	32.117	29.052	**25.004**
			0.9	70.329	64.855	66.992	59.633	52.377	43.55	**33.099**
			0.95	136.42	116.93	125.3	99.129	77.3	54.93	**33.521**
			0.99	666.21	351.5	527.66	146.85	46.828	20.17	**2.8403**
0.6	0.2	3	0.8	3.3789	3.1035	3.0254	2.8414	2.7459	2.8569	**2.3338**
			0.9	6.3296	5.406	5.1772	4.5647	4.2816	4.6497	**3.1983**
			0.95	12.278	9.1533	8.5534	6.5421	5.7885	6.9701	**3.4784**
			0.99	59.959	20.078	19.093	3.3132	2.3913	7.675	**0.5193**
		5	0.8	9.3859	8.6207	8.4038	7.8928	7.6275	7.9359	**6.4825**
			0.9	17.582	15.017	14.381	12.68	11.893	12.916	**8.8839**
			0.95	34.106	25.426	23.759	18.172	16.079	19.361	**9.662**
			0.99	166.55	55.772	53.037	9.2035	6.6426	21.319	**1.4423**
		10	0.8	37.544	34.483	33.615	31.571	30.51	31.744	**25.93**
			0.9	70.329	60.067	57.524	50.719	47.573	51.664	**35.535**
			0.95	136.42	101.7	95.037	72.689	64.316	77.444	**38.647**
			0.99	666.21	223.09	212.15	36.814	26.57	85.278	**5.7692**
	0.5	3	0.8	3.3789	3.1035	3.1553	2.8414	2.6094	2.7343	**2.1292**
			0.9	6.3296	5.406	5.5941	4.5647	3.8921	4.2755	**2.7016**
			0.95	12.278	9.1533	9.8629	6.5421	4.8238	5.9046	**2.5544**
			0.99	59.959	20.078	31.601	3.3132	2.4148	3.055	**0.1571**
		5	0.8	9.3859	8.6207	8.7648	7.8928	7.2481	7.5951	**5.9139**
			0.9	17.582	15.017	15.539	12.68	10.811	11.876	**7.5043**
			0.95	34.106	25.426	27.397	18.172	13.399	16.402	**7.095**
			0.99	166.55	55.772	87.781	9.2035	6.7076	8.486	**0.4363**
		10	0.8	37.544	34.483	35.059	31.571	28.992	30.38	**23.655**
			0.9	70.329	60.067	62.157	50.719	43.245	47.505	**30.017**
			0.95	136.42	101.7	109.59	72.689	53.597	65.606	**28.379**
			0.99	666.21	223.09	351.12	36.814	26.831	33.944	**1.7452**
	0.8	3	0.8	3.3789	3.1035	3.2884	2.8414	2.4803	2.6147	**1.9431**
			0.9	6.3296	5.406	6.0293	4.5647	3.5405	3.9195	**2.2842**
			0.95	12.278	9.1533	11.277	6.5421	4.0243	4.9438	**1.8831**
			0.99	59.959	20.078	47.489	3.3132	3.3704	1.8153	**0.1319**
		5	0.8	9.3859	8.6207	9.1346	7.8928	6.8897	7.263	**5.3967**
			0.9	17.582	15.017	16.748	12.68	9.8346	10.887	**6.3445**
			0.95	34.106	25.426	31.325	18.172	11.178	13.733	**5.2301**
			0.99	166.55	55.772	131.91	9.2035	9.3621	5.0426	**0.3661**
		10	0.8	37.544	34.483	36.538	31.571	27.558	29.052	**21.586**
			0.9	70.329	60.067	66.992	50.719	39.338	43.55	**25.378**
			0.95	136.42	101.7	125.3	72.689	44.713	54.93	**20.919**
			0.99	666.21	223.09	527.66	36.814	37.448	20.17	**1.464**
0.9	0.2	3	0.8	3.3789	2.9798	3.0254	2.6094	2.4803	2.8569	**2.1137**
			0.9	6.3296	5.0261	5.1772	3.8921	3.5405	4.6497	**2.6674**
			0.95	12.278	8.0565	8.5534	4.8238	4.0243	6.9701	**2.492**
			0.99	59.959	14.03	19.093	2.4148	3.3704	7.675	**0.2829**
		5	0.8	9.3859	8.2772	8.4038	7.2481	6.8897	7.9359	**5.8708**
			0.9	17.582	13.962	14.381	10.811	9.8346	12.916	**7.4091**
			0.95	34.106	22.379	23.759	13.399	11.178	19.361	**6.9217**
			0.99	166.55	38.973	53.037	6.7076	9.3621	21.319	**0.7855**
		10	0.8	37.544	33.109	33.615	28.992	27.558	31.744	**23.482**

			0.9	70.329	55.846	57.524	43.245	39.338	51.664	**29.636**
			0.95	136.42	89.516	95.037	53.597	44.713	77.444	**27.686**
			0.99	666.21	155.89	212.15	26.831	37.448	85.278	**3.142**
	0.5	3	0.8	3.3789	2.9798	3.1553	2.6094	2.2999	2.7343	**1.8846**
			0.9	6.3296	5.0261	5.5941	3.8921	3.0752	4.2755	**2.1649**
			0.95	12.278	8.0565	9.8629	4.8238	3.0709	5.9046	**1.7116**
			0.99	59.959	14.03	31.601	2.4148	5.6014	3.055	**0.0737**
		5	0.8	9.3859	8.2772	8.7648	7.2481	6.3883	7.5951	**5.2342**
			0.9	17.582	13.962	15.539	10.811	8.5421	11.876	**6.013**
			0.95	34.106	22.379	27.397	13.399	8.53	16.402	**4.7538**
			0.99	166.55	38.973	87.781	6.7076	15.559	8.486	**0.2044**
		10	0.8	37.544	33.109	35.059	28.992	25.553	30.38	**20.936**
			0.9	70.329	55.846	62.157	43.245	34.168	47.505	**24.052**
			0.95	136.42	89.516	109.59	53.597	34.119	65.606	**19.014**
			0.99	666.21	155.89	351.12	26.831	62.237	33.944	**0.8175**
	0.8	3	0.8	3.3789	2.9798	3.2884	2.6094	2.1338	2.6147	**1.6822**
			0.9	6.3296	5.0261	6.0293	3.8921	2.6741	3.9195	**1.7624**
			0.95	12.278	8.0565	11.277	4.8238	2.3469	4.9438	**1.1892**
			0.99	59.959	14.03	47.489	2.4148	8.1754	1.8153	**0.2111**
		5	0.8	9.3859	8.2772	9.1346	7.2481	5.9267	7.263	**4.6714**
			0.9	17.582	13.962	16.748	10.811	7.4278	10.887	**4.8948**
			0.95	34.106	22.379	31.325	13.399	6.5188	13.733	**3.3022**
			0.99	166.55	38.973	131.91	6.7076	22.709	5.0426	**0.5859**
		10	0.8	37.544	33.109	36.538	28.992	23.706	29.052	**18.684**
			0.9	70.329	55.846	66.992	43.245	29.711	43.55	**19.578**
			0.95	136.42	89.516	125.3	53.597	26.074	54.93	**13.207**
			0.99	666.21	155.89	527.66	26.831	90.837	20.17	**2.343**

NOTE: Minimum MSE value is bolded in each row.

Table 4: Estimated MSE when n=100 and p=7

K	d	sigma	rho	OLS	RIDGE	LIU	K-L	DK	ML	BTP
0.3	0.2	3	0.8	1.5582	1.5296	1.4842	1.5013	1.4901	1.448	**1.3851**
			0.9	2.9335	2.8321	2.6781	2.7326	2.6942	2.555	**2.3483**
			0.95	5.7032	5.3277	4.8022	4.9658	4.8307	4.3823	**3.7215**
			0.99	27.908	20.44	14.275	14.191	12.39	9.2139	**4.2782**
		5	0.8	4.3282	4.2488	4.1228	4.1701	4.1393	4.0221	**3.8474**
			0.9	8.1487	7.867	7.4391	7.5906	7.484	7.0971	**6.5231**
			0.95	15.842	14.799	13.34	13.794	13.419	12.173	**10.337**
			0.99	77.524	56.777	39.653	39.418	34.416	25.594	**11.884**
		10	0.8	17.313	16.995	16.491	16.681	16.557	16.088	**15.39**
			0.9	32.595	31.468	29.756	30.363	29.936	28.389	**26.092**
			0.95	63.369	59.197	53.358	55.175	53.674	48.693	**41.35**
			0.99	310.09	227.11	158.61	157.67	137.66	102.38	**47.535**
	0.5	3	0.8	1.5582	1.5296	1.5117	1.5013	1.4736	1.4211	**1.3445**
			0.9	2.9335	2.8321	2.7724	2.7326	2.6377	2.4647	**2.2188**
			0.95	5.7032	5.3277	5.1305	4.9658	4.6351	4.0808	**3.3305**
			0.99	27.908	20.44	18.839	14.191	10.09	6.1861	**2.4333**
		5	0.8	4.3282	4.2488	4.1992	4.1701	4.0934	3.9474	**3.7347**
			0.9	8.1487	7.867	7.7012	7.5906	7.3269	6.8463	**6.1632**
			0.95	15.842	14.799	14.251	13.794	12.875	11.336	**9.2513**

			0.99	77.524	56.777	52.33	39.418	28.028	17.184	**6.759**
		10	0.8	17.313	16.995	16.797	16.681	16.374	15.79	**14.939**
			0.9	32.595	31.468	30.805	30.363	29.308	27.385	**24.653**
			0.95	63.369	59.197	57.006	55.175	51.501	45.342	**37.005**
			0.99	310.09	227.11	209.32	157.67	112.11	68.735	**27.036**
	0.8	3	0.8	1.5582	1.5296	1.5395	1.5013	1.4573	1.3945	**1.305**
			0.9	2.9335	2.8321	2.8685	2.7326	2.5824	2.3761	**2.0954**
			0.95	5.7032	5.3277	5.4703	4.9658	4.4477	3.7908	**2.9748**
			0.99	27.908	20.44	24.061	14.191	8.1931	3.8168	**1.3008**
		5	0.8	4.3282	4.2488	4.2764	4.1701	4.0481	3.8735	**3.6248**
			0.9	8.1487	7.867	7.9681	7.5906	7.1733	6.6003	**5.8204**
			0.95	15.842	14.799	15.195	13.794	12.355	10.53	**8.2633**
			0.99	77.524	56.777	66.837	39.418	22.759	10.602	**3.6132**
		10	0.8	17.313	16.995	17.105	16.681	16.192	15.494	**14.499**
			0.9	32.595	31.468	31.872	30.363	28.693	26.401	**23.282**
			0.95	63.369	59.197	60.781	55.175	49.419	42.12	**33.053**
			0.99	310.09	227.11	267.35	157.67	91.035	42.409	**14.453**
0.6	0.2	3	0.8	1.5582	1.5018	1.4842	1.4466	1.4253	1.448	**1.3251**
			0.9	2.9335	2.7362	2.6781	2.5462	2.4754	2.555	**2.1592**
			0.95	5.7032	4.99	4.8022	4.3272	4.0959	4.3823	**3.1636**
			0.99	27.908	15.687	14.275	7.1158	5.3331	9.2139	**1.9372**
		5	0.8	4.3282	4.1717	4.1228	4.0182	3.9591	4.0221	**3.6808**
			0.9	8.1487	7.6006	7.4391	7.0728	6.8761	7.0971	**5.9978**
			0.95	15.842	13.861	13.34	12.02	11.377	12.173	**8.7878**
			0.99	77.524	43.574	39.653	19.766	14.814	25.594	**5.381**
		10	0.8	17.313	16.687	16.491	16.073	15.836	16.088	**14.723**
			0.9	32.595	30.402	29.756	28.291	27.504	28.389	**23.991**
			0.95	63.369	55.444	53.358	48.079	45.51	48.693	**35.151**
			0.99	310.09	174.3	158.61	79.065	59.257	102.38	**21.524**
	0.5	3	0.8	1.5582	1.5018	1.5117	1.4466	1.394	1.4211	**1.2723**
			0.9	2.9335	2.7362	2.7724	2.5462	2.373	2.4647	**1.9985**
			0.95	5.7032	4.99	5.1305	4.3272	3.7723	4.0808	**2.7219**
			0.99	27.908	15.687	18.839	7.1158	3.3905	6.1861	**0.9046**
		5	0.8	4.3282	4.1717	4.1992	4.0182	3.8721	3.9474	**3.5341**
			0.9	8.1487	7.6006	7.7012	7.0728	6.5917	6.8463	**5.5514**
			0.95	15.842	13.861	14.251	12.02	10.479	11.336	**7.5609**
			0.99	77.524	43.574	52.33	19.766	9.4181	17.184	**2.5127**
		10	0.8	17.313	16.687	16.797	16.073	15.488	15.79	**14.136**
			0.9	32.595	30.402	30.805	28.291	26.367	27.385	**22.205**
			0.95	63.369	55.444	57.006	48.079	41.914	45.342	**30.243**
			0.99	310.09	174.3	209.32	79.065	37.672	68.735	**10.051**
	0.8	3	0.8	1.5582	1.5018	1.5395	1.4466	1.3634	1.3945	**1.2216**
			0.9	2.9335	2.7362	2.8685	2.5462	2.275	2.3761	**1.8492**
			0.95	5.7032	4.99	5.4703	4.3272	3.4744	3.7908	**2.3385**
			0.99	27.908	15.687	24.061	7.1158	2.0921	3.8168	**0.4009**
		5	0.8	4.3282	4.1717	4.2764	4.0182	3.7872	3.8735	**3.393**
			0.9	8.1487	7.6006	7.9681	7.0728	6.3194	6.6003	**5.1366**
			0.95	15.842	13.861	15.195	12.02	9.6511	10.53	**6.4957**
			0.99	77.524	43.574	66.837	19.766	5.8112	10.602	**1.1136**
		10	0.8	17.313	16.687	17.105	16.073	15.149	15.494	**13.572**
			0.9	32.595	30.402	31.872	28.291	25.278	26.401	**20.546**
			0.95	63.369	55.444	60.781	48.079	38.604	42.12	**25.983**

			0.99	310.09	174.3	267.35	79.065	23.245	42.409	**4.4544**
0.9	0.2	3	0.8	1.5582	1.4748	1.4842	1.394	1.3634	1.448	**1.2679**
			0.9	2.9335	2.6454	2.6781	2.373	2.275	2.555	**1.9859**
			0.95	5.7032	4.6849	4.8022	3.7723	3.4744	4.3823	**2.6906**
			0.99	27.908	12.451	14.275	3.3905	2.0921	9.2139	**0.8106**
		5	0.8	4.3282	4.0968	4.1228	3.8721	3.7872	4.0221	**3.5218**
			0.9	8.1487	7.3482	7.4391	6.5917	6.3194	7.0971	**5.5164**
			0.95	15.842	13.014	13.34	10.479	9.6511	12.173	**7.4739**
			0.99	77.524	34.587	39.653	9.4181	5.8112	25.594	**2.2515**
		10	0.8	17.313	16.387	16.491	15.488	15.149	16.088	**14.087**
			0.9	32.595	29.393	29.756	26.367	25.278	28.389	**22.065**
			0.95	63.369	52.055	53.358	41.914	38.604	48.693	**29.896**
			0.99	310.09	138.35	158.61	37.672	23.245	102.38	**9.0059**
	0.5	3	0.8	1.5582	1.4748	1.5117	1.394	1.3189	1.4211	**1.2042**
			0.9	2.9335	2.6454	2.7724	2.373	2.1358	2.4647	**1.8009**
			0.95	5.7032	4.6849	5.1305	3.7723	3.0708	4.0808	**2.2253**
			0.99	27.908	12.451	18.839	3.3905	0.9662	6.1861	**0.291**
		5	0.8	4.3282	4.0968	4.1992	3.8721	3.6634	3.9474	**3.3449**
			0.9	8.1487	7.3482	7.7012	6.5917	5.9326	6.8463	**5.0023**
			0.95	15.842	13.014	14.251	10.479	8.53	11.336	**6.1812**
			0.99	77.524	34.587	52.33	9.4181	2.684	17.184	**0.8083**
		10	0.8	17.313	16.387	16.797	15.488	14.654	15.79	**13.379**
			0.9	32.595	29.393	30.805	26.367	23.73	27.385	**20.009**
			0.95	63.369	52.055	57.006	41.914	34.12	45.342	**24.725**
			0.99	310.09	138.35	209.32	37.672	10.736	68.735	**3.2329**
	0.8	3	0.8	1.5582	1.4748	1.5395	1.394	1.2759	1.3945	**1.1438**
			0.9	2.9335	2.6454	2.8685	2.373	2.0052	2.3761	**1.6328**
			0.95	5.7032	4.6849	5.4703	3.7723	2.7132	3.7908	**1.838**
			0.99	27.908	12.451	24.061	3.3905	0.4889	3.8168	**0.1014**
		5	0.8	4.3282	4.0968	4.2764	3.8721	3.5439	3.8735	**3.1768**
			0.9	8.1487	7.3482	7.9681	6.5917	5.5699	6.6003	**4.5354**
			0.95	15.842	13.014	15.195	10.479	7.5367	10.53	**5.1055**
			0.99	77.524	34.587	66.837	9.4181	1.3581	10.602	**0.2815**
		10	0.8	17.313	16.387	17.105	15.488	14.176	15.494	**12.707**
			0.9	32.595	29.393	31.872	26.367	22.279	26.401	**18.141**
			0.95	63.369	52.055	60.781	41.914	30.147	42.12	**20.422**
			0.99	310.09	138.35	267.35	37.672	5.4322	42.409	**1.1258**

NOTE: Minimum MSE value is bolded in each row.

Simulation results discussions

Table 1-4 shows that when σ, ρ, and p increase, the estimated MSE values also increase. In addition, the MSE decreases as the sample size *n* increase. The OLS estimator performs the worst among all the estimators used in this study which is expected due to multicollinearity. Also, we observe that the proposed BTP estimator dominates other estimators in this study. Thus, the findings agree with the theoretical results. For each row, the smallest MSE value is bolded.

5. Application

In this section, Portland cement data was used to demonstrate the performance of the proposed estimator. The Portland cement data was originally adopted by Woods *et al*. [32] and was later adopted by other authors such as Kaciranlar *et al.* [20], Ayinde *et al*. [33], and Lukman *et al*. [34]. The data set is widely known as the Portland cement

Table 5. Results of regression coefficients and the MSE values for Portland Cement Data

Coef	OLS	RIDGE	LIU	KL	ML	DK	BTP
$\hat{\beta}_1$	-52.9936	-50.5173	-50.9877	-48.041	-46.9760	42.3386	-45.7216
$\hat{\beta}_2$	0.071073	0.0706	0.0707	0.0700	0.0698	43.9155	0.0696
$\hat{\beta}_3$	-0.4142	-0.41	-0.4107	-0.4056	-0.4037	12.7444	-0.4015
$\hat{\beta}_4$	-0.42347	-0.4295	-0.4283	-0.4354	-0.4380	0.8349	-0.4411
$\hat{\beta}_5$	-0.57257	-0.574	-0.5737	-0.5754	-0.576	0.3622	-0.5767
$\hat{\beta}_6$	48.41787	48.1434	48.1951	47.869	47.75	-4.8196	47.6109
MSE	17095.15	15669.12	15934.2	14318.66	13716.23	13122.63	**10605.56**
Kld		0.6	0.5	0.6	0.5	0.6/0.5	0.6/0.5

dataset. The regression model for these data is defined as follows:

$$y_i = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \varepsilon_i \tag{47}$$

where y_i = heat evolved after 180 days of curing measured in calories per gram of cement, X_1 = tricalcium aluminate, X_2 = tricalcium silicate, X_3 = tetracalcium aluminoferrite, and $X_4 = \beta$-dicalcium silicate. The variance inflation factors are VIF1 = 38.50, VIF2 = 254.42, VIF3 = 46.87, and VIF4 = 282.51. Eigenvalues of $X'X$ matrix are λ1 = 44676.206, λ2 = 5965.422, λ3 = 809.952, and λ4 = 105.419, and the condition number of $X'X$ is approximately 424. The VIFs, eigenvalues, and condition numbers indicate that severe multicollinearity exists. The estimated parameters and the MSE values of the estimators are presented in Table 5. It is clearly shown and obvious from Table 5 that the proposed BTP estimator dominates all other considered estimators in this study.

6. Conclusion

This paper proposes a new biased regression estimator called BTP as an alternative to OLS to address the problem of multicollinearity in the linear regression model. We theoretically obtained the biasing parameters of the proposed and compared this new estimation method with some existing estimators (OLS, ORR, LIU, KL, ML, and DK). The simulation study and the theoretical findings revealed the superiority of the proposed method. Likewise, real-life data was used and analyzed to support the theoretical finding and simulation results. Furthermore, it can be seen that the performance of the estimators depends on the biasing parameter.

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