Published by
NIGERIAN SOCIETY OF PHYSICAL SCIENCES
Available online @ https://asr.nsps.org.ng/index.php/asr

African Scientific Reports 5 (2026) 491

African Scientific Reports

Development of the hesitant fuzzy weighted averaging–geometric aggregation operator (HFW-AG) under hesitant fuzzy information

Rafi Raza[a], Ahmad Termimi Bin Ab Ghani[a,*], Iraq T. Abbas[b], Lazim Abdullah[a]

[a]*Faculty of Computer Science and Mathematics, University of Malaysia Terengganu, Kuala Nerus 210300, Malaysia*
[b]*College of Science, Department of Mathematics, University of Baghdad, Baghdad 10071, Iraq*

Abstract

The hesitant fuzzy set (HFS) framework has been widely used in multi-criteria group decision-making (MCGDM) to represent uncertainty and hesitation in decision-makers' assessments. However, existing hesitant fuzzy aggregation operators often rely solely on either averaging or geometric aggregation mechanisms, which may limit their ability to capture different characteristics of hesitant fuzzy information. To address this issue, this study proposes a new hesitant fuzzy weighted averaging–geometric aggregation (HFW-AG) operator that combines the features of weighted averaging and geometric aggregation within a unified framework. Several mathematical properties of the proposed operator, including boundedness, monotonicity, and idempotency, are established to demonstrate its theoretical validity. Furthermore, an MCGDM procedure based on the HFW-AG operator is developed for decision-making under hesitant fuzzy environments. A numerical example is presented to illustrate the implementation of the proposed approach, and comparative analyses are conducted with the existing hesitant fuzzy weighted averaging (HFWA) and hesitant fuzzy weighted geometric (HFWG) operators. The results show that the HFW-AG operator exhibits a distinct aggregation behavior and produces a different ranking pattern of alternatives, reflecting its ability to capture hesitant fuzzy information from both additive and multiplicative perspectives. These findings demonstrate the applicability of the proposed operator and its potential usefulness for MCGDM problems involving uncertainty and ambiguity.

DOI: 10.46481/asr.2026.5.2.491

Keywords: Hesitant fuzzy sets, Multi-criteria group decision-making, Aggregation operator, HFW-AG operator

Article History:
Received: 24 March 2026
Received in revised form: 05 July 2026
Accepted for publication: 14 July 2026
Available online: 05 August 2026

1. Introduction

Decision-making is a fundamental process in management that involves selecting the most appropriate alternative from a set of available options. In today's globalized and highly competitive environment, managerial decisions are increasingly influenced by cross-border activities, internationalization, and globalization trends [1, 2]. All problems related to such choices must be addressed because the effectiveness and efficiency of decision-making processes may be directly affected by this task [1]. Furthermore, precisely evaluating expert viewpoints in real-world choice problems makes it challenging to meet the requirements for handling complicated decision-making concerns [3].

*Corresponding author Tel. No.: +92-344-5356053
Email address: termimi@umt.edu.my (Ahmad Termimi Bin Ab Ghani)

By explicitly assessing feasible solutions against competing criteria, multiple-criteria decision-making (MCDM) uses advanced mathematical tools to identify an optimal course of action [4]. It is typically impossible for one person to thoroughly consider all aspects of a topic because of the growing complexity of choice scenarios, which makes group decision-making (GDM) vital [5]. GDM involves several people working together to analyze problems, assess potential solutions that satisfy a variety of conflicting requirements, and select an appropriate option [4]. As a result, several multi-criteria group decision-making (MCGDM) approaches have been created to account for the viewpoints of managers and decision-makers (DMs) with varying levels of expertise and specialization [6, 7].

MCGDM is a well-known process for selecting an ideal item when experts evaluate options based on multiple criteria. GDM can enhance evaluation by weighing different and sometimes conflicting information. Because criteria represent the effectiveness of choices from different aspects, uncertainty in DMs' judgments is difficult to avoid. Scholars have studied the DM process worldwide and have produced successful results using various methods. Zadeh [8] developed the well-known notion of fuzzy sets (FSs), which describes ambiguous information in MCGDM situations and helps address hazy and doubtful data. Following the creation of FSs, researchers interpreted and extended this concept in different ways.

Numerous extensions, including intuitionistic fuzzy sets (IFSs) [9], type-2 fuzzy sets [10, 11], type-n fuzzy sets [10], fuzzy multisets [12, 13], and HFSs [14, 15], have been developed since the introduction of FSs [8]. HFSs allow a member to have a range of possible values and are applicable to a wide range of decision-making scenarios. In a decision-making situation that includes numerous criteria and multiple participants, there are typically two approaches for determining the optimal choice: first, combining the DMs' viewpoints under each alternative's criterion and then combining the overall values of those criteria for every option; and second, combining the DMs' criteria values for every option and then combining the DMs' viewpoints for every option. Anonymity is necessary in many real-world scenarios, such as presidential elections or blind peer review, to safeguard DMs' privacy or prevent them from influencing one another. When it is not known which criterion each DM is familiar with, all options must be weighed to support rational conclusions. However, current approaches usually consider only limited scenarios in which each DM is proficient at assessing every criterion. By using HFSs, which allow each option to be explained in an HFS defined based on the opinions of DMs [15], and by using HFEs to represent each criterion under the alternative, such issues can be addressed more effectively. The HFEs for each alternative should then be fused using aggregation approaches according to the requirements.

Human knowledge may be represented either as exact crisp information or as imprecise fuzzy information. A finite set of numerical values can be combined and summarized into a single numerical value using aggregation operators [17]. MCGDM problems require the use of these operators. Generally, MCGDM problems involve a finite set of choices from which the best option must be selected based on a finite set of criteria. Experts are contacted to provide a value or score for each criterion element of the alternative. The extent to which an alternative satisfies a criterion may be interpreted as an individual's choice or opinion. Each expert provides a judgment on the same scale, either qualitative or quantitative, for each criterion. According to Marichal [18], the three main steps of the MCGDM procedure are modeling, aggregation, and exploitation.

Decision-making theory is critical in foreign policy, national defense policy, economic policy, medical diagnosis, and related fields. Decision-making problems have mainly used three tools: correlation coefficients, similarity measures, and aggregation operators. Real-world decision-making situations usually involve ambiguity and uncertainty, making them challenging to resolve using conventional techniques. Various techniques have been devised to address these issues, and a review of the literature indicates that many scholars have examined decision-making problems based on different types of FSs, including correlation coefficient processes [19, 20] and similarity measures [21, 22]. Şenel *et al*. [23] developed similarity metrics between octahedron sets, previously proposed by Lee *et al*. [24], and utilized them in MCGDM problems.

One major issue is the difficulty of using fuzzy aggregation operators. Xia and Xu [16] first presented hesitant fuzzy weighted averaging (HFWA) and hesitant fuzzy weighted geometric (HFWG) methods to address decision-making challenges. Xu and Chen [25] created an interval-valued intuitionistic fuzzy Bonferroni mean operator (BMO) for MCDM. Liu *et al*. [26] addressed multiple-attribute group decision-making (MAGDM) difficulties by proposing a BMO based on IFSs. Garg and Arora [27] presented an aggregation BMO under intuitionistic fuzzy soft sets for decision-making scenarios. Kaur and Garg [28] presented an aggregation BMO on cubic intuitionistic fuzzy sets for MAGDM. Conversely, several aggregation procedures for interval-valued intuitionistic hesitant fuzzy sets were used by Zhang [29] to perform GDM. Sarwar *et al*. [33] proposed a two-sided matching optimization model based on hesitant 2-tuple linguistic rough numbers for green housing technology selection. Akram *et al*. [34] developed complex Pythagorean fuzzy Yager aggregation operators and an MCDM model. Sarkar *et al*. [35] developed a hybrid MCGDM approach based on dual hesitant q-rung orthopair fuzzy Frank power-partitioned Heronian mean aggregation operators. Akram *et al*. [36] proposed complex intuitionistic fuzzy Hamacher aggregation operators and developed a multiple-attribute decision-making (MADM) model. Ning *et al*. [37] proposed probabilistic spherical hesitant fuzzy sets and developed generalized weighted averaging and geometric operators for MADM. Kirişci *et al*. [38] proposed a hesitant fuzzy decision-making method with aggregation operators for investment portfolio selection under interval-valued Fermatean hesitant fuzzy sets. Ma and Zeng [39] introduced Bonferroni mean operators on weighted hesitant fuzzy sets for multi-criteria symmetric group decision-making.

The proposed HFW-AG operator is fundamentally different from the existing HFWA and HFWG operators. The HFWA operator captures additive behavior through arithmetic aggregation, whereas the HFWG operator reflects multiplicative behavior through geometric aggregation. However, each operator is limited to a single aggregation mechanism, which restricts its ability to fully represent complex hesitant fuzzy information. To address this limitation, the HFW-AG operator integrates both additive and multiplicative aggregation behaviors within a unified mathematical structure. This formulation is motivated by the fact that many real-world decision-making problems involve criteria exhibiting both compensatory and proportional characteristics. Therefore, the proposed operator fills a theoretical gap by providing a more comprehensive aggregation tool for uncertain hesitant fuzzy environments.

This study introduces the HFW-AG operator for HFSs in MCGDM problems. The proposed operator unifies additive and multiplicative aggregation behaviors within a single framework, enabling more effective handling of complex hesitant fuzzy information. Furthermore, this study provides an overview of existing hesitant fuzzy aggregation operators, analyzes the basic properties of the proposed operator, and presents illustrative examples to demonstrate its computational procedure and effectiveness in MCGDM settings.

The paper is organized as follows. Section 2 presents the preliminaries and basic concepts related to the study. Section 3 introduces the hesitant fuzzy aggregation operators employed in the proposed framework. Section 4 describes the MCGDM approach under hesitant fuzzy information. Section 4.1 presents the numerical example, whereas Section 4.2 provides the discussion and analysis of the obtained results. Finally, Section 5 concludes the paper and outlines potential directions for future research.

2. Preliminaries

Definition 2.1. Let X be the universal set. A fuzzy set B in X is defined by the membership function $\mu(x) : X \to [0, 1]$ and can be written as

$$B = \{(x, \mu(x)) \mid x \in X\}. \tag{1}$$

In Eq. (1), $\mu(x)$ is the degree of membership of x in B, and each pair $(x, \mu(x))$ is a singleton [28].

Definition 2.2. Suppose that X is the universal set. An HFS B on X is defined by the function $\rho_B(\widetilde{x})$ such that X returns to a subset of $[0, 1]$, as follows:

$$B = \{\langle \widetilde{x}, \rho_B(\widetilde{x})\rangle \mid \widetilde{x} \in X\}. \tag{2}$$

In Eq. (2), $\rho_B(\widetilde{x})$ is defined as a set of membership degrees for an element under a subset of $[0, 1]$, and the membership degree of an element B is a subset of X [14, 15].

Example 1. Suppose $X = \{\widetilde{l}, \widetilde{m}, \widetilde{n}\}$ is a nonempty set, with

$$\rho_B(\widetilde{l}) = \{0.3, 0.9, 0.6\}, \quad \rho_B(\widetilde{m}) = \{0.6, 0.7\}, \quad \rho_B(\widetilde{n}) = \{0.7, 0.3\}.$$

Then the HFS can be written as

$$B = \{\langle \widetilde{l}, \{0.3, 0.9, 0.6\}\rangle, \langle \widetilde{m}, \{0.6, 0.7\}\rangle, \langle \widetilde{n}, \{0.7, 0.3\}\rangle\}.$$

Definition 2.3. For an HFE h, the score function is defined as

$$s(h) = \frac{1}{l_h} \sum_{\gamma \in h} \gamma. \tag{3}$$

In Eq. (3), l_h is the number of elements in h. Additionally, for two HFEs h_1 and h_2, if $s(h_1) > s(h_2)$, then h_1 is greater than h_2, denoted by $h_1 > h_2$. If $s(h_1) = s(h_2)$, then h_1 is indifferent to h_2, denoted by $h_1 \sim h_2$ [30].

Example 2. Let $h_1 = \{0.5, 0.6, 0.8\}$, $h_2 = \{0.5, 0.2\}$, and $h_3 = \{0.8, 0.9\}$. Using Eq. (3), we obtain

$$s(h_1) = \frac{0.5 + 0.6 + 0.8}{3} = 0.6333, \quad s(h_2) = \frac{0.5 + 0.2}{2} = 0.3500, \quad s(h_3) = \frac{0.8 + 0.9}{2} = 0.8500.$$

Thus, $s(h_3) > s(h_1) > s(h_2)$. Hence, $h_3 > h_1 > h_2$, which shows that h_3 is the most preferred alternative, followed by h_1, while h_2 is the least preferred alternative.

3. Aggregation operator

Aggregation operators play an important role in combining multiple HFEs into a single representative value for decision-making. These operators help transform a set of individual evaluations into a meaningful collective assessment. In this section, we first present two well-known hesitant fuzzy aggregation operators, namely the HFWA operator in Eq. (4) and the HFWG operator in Eq. (5). Based on these operators, the proposed HFW-AG operator in Eq. (6) is introduced. Finally, the relationships among these operators are established through several mathematical results.

Definition 3.1. Suppose $h_1, h_2, \ldots, h_n$ are a group of HFEs. The HFWA operator is a mapping $H^n \to H$ such that

$$\mathrm{HFWA}(h_1, h_2, \ldots, h_n) = \bigcup_{\gamma_1 \in h_1, \ldots, \gamma_n \in h_n} \left\{ 1 - \prod_{j=1}^{n} (1 - \gamma_j)^{w_j} \right\}. \tag{4}$$

Definition 3.2. Suppose $h_1, h_2, \ldots, h_n$ are a group of HFEs. The HFWG operator is a mapping $H^n \to H$ such that

$$\mathrm{HFWG}(h_1, h_2, \ldots, h_n) = \bigcup_{\gamma_1 \in h_1, \ldots, \gamma_n \in h_n} \left\{ \prod_{j=1}^{n} \gamma_j^{w_j} \right\}. \tag{5}$$

Definition 3.3. Although HFWA and HFWG operators are widely used in hesitant fuzzy decision-making, each captures only a single type of aggregation behavior. To overcome this limitation, the HFW-AG operator is proposed as a unified aggregation mechanism that incorporates both averaging and geometric characteristics for processing hesitant fuzzy information. Suppose $h_1, h_2, \ldots, h_n$ are a group of HFEs. The HFW-AG operator is a mapping $H^n \to H$ such that

$$\mathrm{HFW\text{-}AG}(h_1, h_2, \ldots, h_n) = \bigcup_{\gamma_1 \in h_1, \ldots, \gamma_n \in h_n} \left\{ 1 - \prod_{j=1}^{n} [\gamma_j (1 - \gamma_j)]^{w_j} \right\}. \tag{6}$$

The proposed operator in Eq. (6) integrates both averaging and geometric aggregation characteristics within a single formulation, enabling a more comprehensive representation of hesitant fuzzy information compared with existing HFWA and HFWG operators.

3.1. Properties of the proposed HFW-AG operator

Let h_j $(j = 1, \ldots, n)$ be a collection of HFEs, and let $w = (w_1, \ldots, w_n)^T$ be the associated weighting vector satisfying $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$. Based on Eq. (6), the proposed HFW-AG operator has the following properties:

1. If all h_j are equal, i.e., $h_j = h$ for all j, then

$$\mu_{\text{HFW-AG}}\left(h_i \ (i = 1, \ldots, n)\right) = h,$$

which gives idempotency.

2. Let h_j and h'_j $(j = 1, \ldots, n)$ be two collections of HFEs. If $h_j \leq h'_j$ for all j, then

$$\mu_{\text{HFW-AG}}\left(h_i \ (i = 1, \ldots, n)\right) \leq \mu_{\text{HFW-AG}}\left(h'_i \ (i = 1, \ldots, n)\right),$$

which gives monotonicity.

3. Let h_j $(j = 1, \ldots, n)$ be any permutation of h'_j $(j = 1, \ldots, n)$. Then

$$\mu_{\text{HFW-AG}}\left(h_i \ (i = 1, \ldots, n)\right) = \mu_{\text{HFW-AG}}\left(h'_i \ (i = 1, \ldots, n)\right),$$

which gives commutativity.

4. Let $h_j^+ = \max_j h_j$ and $h_j^- = \min_j h_j$. Then

$$h_j^- \leq \mu_{\text{HFW-AG}}\left(h_j \ (j = 1, \ldots, n)\right) \leq h_j^+,$$

which gives boundedness.

5. If all input HFE values are zero, the result of the aggregation operation is

$$\mu_{\text{HFW-AG}}\left(h_i \ (i = 1, \ldots, n)\right) = 1,$$

which gives the annihilator effect.

3.2. Relationships among operators

The following lemma and theorems establish the relationship among the HFWA, HFWG, and HFW-AG operators.

Lemma 3.1. Let $x_j > 0$ and $\lambda_j > 0$, $j = 1, 2, \ldots, n$, with $\sum_{j=1}^{n} \lambda_j = 1$. Then

$$\prod_{j=1}^{n} x_j^{\lambda_j} \leq \sum_{j=1}^{n} x_j \lambda_j. \tag{7}$$

Equality in Eq. (7) holds if and only if $x_1 = x_2 = \cdots = x_n$ [31, 32].

Theorem 3.1. Let $h_1, h_2, \ldots, h_n$ be a collection of HFEs with weight vector $W = (w_j)^T$ such that $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$. Then

$$\text{HFWG}(h_1, h_2, \ldots, h_n) \leq \text{HFWA}(h_1, h_2, \ldots, h_n). \tag{8}$$

Proof. Let $\gamma_j \in h_j$ for each $j = 1, 2, \ldots, n$. Since HFEs are defined on $[0,1]$, $0 \leq \gamma_j \leq 1$. Hence, all expressions γ_j and $1 - \gamma_j$ are non-negative, ensuring that Lemma 3.1 is applicable.

Using Lemma 3.1 with $x_j = \gamma_j$ and $w_j = \lambda_j$, we obtain

$$\prod_{j=1}^{n} \gamma_j^{w_j} \leq \sum_{j=1}^{n} w_j \gamma_j.$$

Since $\gamma_j \in [0,1]$, $1 - \gamma_j \geq 0$. Applying Lemma 3.1 again with $x_j = 1 - \gamma_j$ gives

$$\prod_{j=1}^{n} (1 - \gamma_j)^{w_j} \leq \sum_{j=1}^{n} w_j (1 - \gamma_j).$$

Multiplying both sides by -1 reverses the inequality and adding 1 to both sides gives the required comparative form. Therefore, Eq. (8) follows. □

Theorem 3.2. Let $h_1, h_2, \ldots, h_n$ be a collection of HFEs with weight vector $W = (w_j)^T$ such that $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$. Then

$$\text{HFWA}(h_1, h_2, \ldots, h_n) \leq \text{HFW-AG}(h_1, h_2, \ldots, h_n). \tag{9}$$

Proof. Since $\gamma_j \in [0,1]$ for all j, all terms in Eqs. (4) and (6) are well-defined. Applying the weighted arithmetic mean–geometric mean inequality to non-negative real numbers yields

$$\prod_{j=1}^{n} \gamma_j^{w_j} \leq \sum_{j=1}^{n} w_j \gamma_j.$$

Similarly, applying the same inequality to $1 - \gamma_j$ yields

$$\prod_{j=1}^{n} (1 - \gamma_j)^{w_j} \leq \sum_{j=1}^{n} w_j (1 - \gamma_j).$$

Because $\gamma_j (1 - \gamma_j) \leq (1 - \gamma_j)$ for $\gamma_j \in [0,1]$, the aggregation form in Eq. (6) produces the comparative relation in Eq. (9). □

Theorem 3.3. Let $h_1, h_2, \ldots, h_n$ be a collection of HFEs with weight vector $W = (w_j)^T$ such that $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$. Then

$$\text{HFWG}(h_1, h_2, \ldots, h_n) \leq \text{HFWA}(h_1, h_2, \ldots, h_n) \leq \text{HFW-AG}(h_1, h_2, \ldots, h_n). \tag{10}$$

Proof. The first inequality in Eq. (10) follows from Theorem 3.1, and the second inequality follows from Theorem 3.2. Therefore, combining the two results gives Eq. (10). □

These results demonstrate the mathematical consistency and comparative behavior of the proposed aggregation framework.

4. MCGDM method under a hesitant fuzzy decision matrix

The MCGDM procedure under hesitant fuzzy information consists of the following steps.

Step 1: Construct the fuzzy decision matrix. The DM defines the relevant criteria for the likely alternatives and gives the alternatives' HFEs in relation to the criteria. In addition, the DM provides ordering weights w_j ($j = 1, 2, \ldots, m$) for several criteria to consider the possibility that distinct alternatives may have different goals and advantages.

Step 2: Use the aggregation operator to obtain the HFEs h_i for the alternatives A_i, where $i = 1, 2, \ldots, n$.

Step 3: Compute the score function $S(h_i)$ for h_i, where $i = 1, 2, \ldots, n$.

Step 4: Rank the alternatives through the values of $S(h_i)$ and obtain the best alternative.

Table 1: Hesitant fuzzy decision matrix.

Alternatives	c_1	c_2	c_3
A_1	$(0.5, 0.6, 0.8)$	$(0.5, 0.7)$	$(0.2, 0.5, 0.8)$
A_2	$(0.6, 0.8)$	$(0.3, 0.4, 0.7, 0.8)$	$(0.3, 0.7)$
A_3	$(0.5, 0.7)$	$(0.5, 0.6, 0.8)$	$(0.2, 0.4, 0.6)$
A_4	$(0.5, 0.7, 0.8)$	$(0.6, 0.8)$	$(0.1, 0.3, 0.6)$

4.1. Numerical example

To demonstrate the applicability and effectiveness of the proposed HFW-AG operator, an MCGDM problem related to selecting an air-conditioning system for a university auditorium hall is considered. The higher authorities of the university intend to install a suitable air-conditioning system that can satisfy economic, functional, and operational requirements under uncertain decision-making conditions.

Suppose that four different air-conditioning systems, denoted by A_1, A_2, A_3, and A_4, are available alternatives for installation in the auditorium hall. The evaluation process is carried out based on three criteria: c_1 (economic), c_2 (functional), and c_3 (operational). The criteria weights are represented by $w = (0.3, 0.2, 0.5)$, where the operational criterion has the highest importance in the decision-making process.

The assessment values are expressed in the form of HFEs to capture hesitation and uncertainty in the opinions of DMs. These hesitant fuzzy values represent possible membership degrees assigned by experts during the evaluation process. The hesitant fuzzy decision matrix $H = (h_{ij})_{4\times3}$ corresponding to the four alternatives and three criteria is presented in Table 1.

Solution. In Step 1, consider the decision matrix presented in Table 1, where $H = (h_{ij})_{4\times3}$ is in the form of HFEs with criteria weight $w = (0.3, 0.2, 0.5)$.

In Step 2, the HFWA, HFWG, and HFW-AG aggregation operators in Eqs. (4)–(6) are used, and the score function of each aggregation operator is calculated.

For the HFWA operator, the aggregated HFEs are

$$\begin{aligned}
h_1 &= \{0.3695, 0.5000, 0.6837, 0.4289, 0.5485, 0.7144, 0.4084, 0.5323, 0.5323,\\
&\quad 0.7042, 0.4659, 0.5777, 0.7329, 0.5195, 0.6201, 0.7597, 0.5662, 0.6570, 0.7831\},\\
h_2 &= \{0.4081, 0.6125, 0.4261, 0.6243, 0.5004, 0.6729, 0.5393, 0.6984,\\
&\quad 0.5192, 0.6853, 0.5338, 0.6948, 0.5942, 0.7343, 0.6258, 0.7550\},\\
h_3 &= \{0.3675, 0.4522, 0.5527, 0.3951, 0.4761, 0.5723, 0.4734, 0.5439,\\
&\quad 0.6276, 0.4574, 0.5300, 0.6163, 0.4810, 0.5506, 0.6330, 0.5482, 0.6087, 0.6805\},\\
h_4 &= \{0.3584, 0.4342, 0.5723, 0.4415, 0.5074, 0.6276, 0.4496, 0.5146,\\
&\quad 0.6330, 0.5208, 0.5774, 0.6805, 0.5126, 0.5701, 0.6750, 0.5757, 0.6258, 0.7171\}.
\end{aligned}$$

For the HFWG operator, the aggregated HFEs are

$$\begin{aligned}
h_1 &= \{0.3162, 0.5000, 0.6324, 0.3382, 0.5348, 0.6764, 0.3340, 0.5281, 0.6680,\\
&\quad 0.3572, 0.5648, 0.7145, 0.3641, 0.5757, 0.7282, 0.3894, 0.6157, 0.7789\},\\
h_2 &= \{0.3693, 0.5641, 0.3912, 0.5975, 0.4375, 0.6683, 0.4493, 0.6864,\\
&\quad 0.4026, 0.6150, 0.4264, 0.6514, 0.4769, 0.7286, 0.4898, 0.7483\},\\
h_3 &= \{0.3162, 0.4472, 0.5477, 0.3279, 0.4638, 0.5680, 0.3473, 0.4912,\\
&\quad 0.6017, 0.3498, 0.4947, 0.6058, 0.3628, 0.5130, 0.6283, 0.3842, 0.5434, 0.6656\},\\
h_4 &= \{0.2319, 0.4016, 0.5680, 0.2456, 0.4254, 0.6017, 0.2565, 0.4443,\\
&\quad 0.6283, 0.2717, 0.4706, 0.6656, 0.2670, 0.4625, 0.6540, 0.2828, 0.4898, 0.6928\}.
\end{aligned}$$

For the HFW-AG operator, the aggregated HFEs are

$$\begin{aligned}
h_1 &= \{0.8001, 0.7500, 0.8000, 0.8069, 0.7586, 0.8069, 0.8025, 0.7531, 0.8025,\\
&\quad 0.8093, 0.7616, 0.8093, 0.8251, 0.7814, 0.8251, 0.8311, 0.7889, 0.8311\},\\
h_2 &= \{0.7815, 0.7815, 0.7756, 0.7756, 0.7815, 0.7815, 0.7930, 0.7930,\\
&\quad 0.8065, 0.8065, 0.8013, 0.8013, 0.8065, 0.8065, 0.8167, 0.8167\},\\
h_3 &= \{0.8000, 0.7551, 0.7551, 0.8017, 0.7571, 0.7571, 0.8171, 0.7760,
\end{aligned}$$

Table 2: Score values and rankings of alternatives obtained by the HFWA, HFWG, and HFW-AG operators.

Aggregation operator	$s(h_1)$	$s(h_2)$	$s(h_3)$	$s(h_4)$	Ranking
HFWA	0.5873	0.6015	0.5314	0.5520	$A_2 > A_1 > A_4 > A_3$
HFWG	0.5342	0.5439	0.4810	0.4477	$A_2 > A_1 > A_3 > A_4$
HFW-AG	0.7968	0.7953	0.7833	0.8137	$A_4 > A_1 > A_2 > A_3$

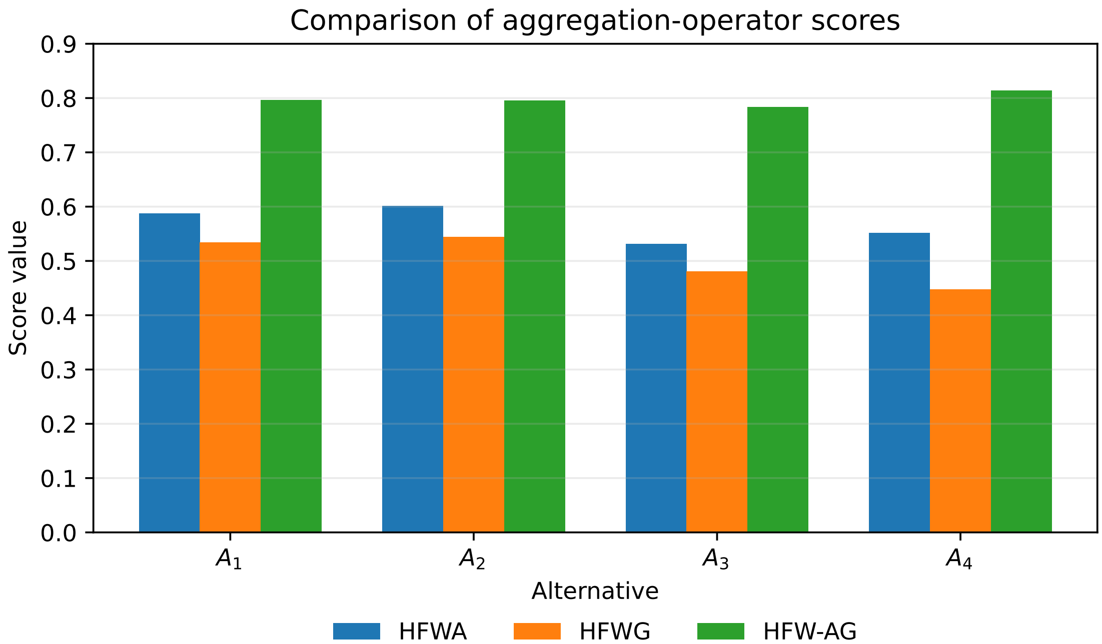

Figure 1: Comparison of the score values produced by the HFWA, HFWG, and HFW-AG aggregation operators.

$$
\begin{aligned}
&0.7760, 0.8171, 0.7676, 0.7676, 0.8118, 0.7695, 0.7695, 0.8264, 0.7874, 0.7874\},\\
h_4 = \{&0.8513, 0.7728, 0.7570, 0.8690, 0.7905, 0.7760, 0.8589, 0.7844,\\
&0.7695, 0.8698, 0.8012, 0.7874, 0.8699, 0.8013, 0.7875, 0.8800, 0.8167, 0.8041\}.
\end{aligned}
$$

In Step 3, the computed scores for the alternatives are obtained using Eq. (3). For the HFWA operator,

$$s(h_1) = \frac{10.5720}{18} = 0.5873, \quad s(h_2) = \frac{9.6244}{16} = 0.6015, \quad s(h_3) = \frac{9.5665}{18} = 0.5314, \quad s(h_4) = \frac{9.9936}{18} = 0.5552.$$

For the HFWG operator,

$$s(h_1) = \frac{9.6166}{18} = 0.5342, \quad s(h_2) = \frac{8.7026}{16} = 0.5439, \quad s(h_3) = \frac{8.6586}{18} = 0.4810, \quad s(h_4) = \frac{8.0601}{18} = 0.4477.$$

For the HFW-AG operator,

$$s(h_1) = \frac{14.3435}{18} = 0.7968, \quad s(h_2) = \frac{12.7252}{16} = 0.7953, \quad s(h_3) = \frac{14.0995}{18} = 0.7833, \quad s(h_4) = \frac{14.6473}{18} = 0.8137.$$

In Step 4, the ranking results of the alternatives are summarized in Table 2. In addition, Figure 1 presents a comparative visualization of the HFWA, HFWG, and HFW-AG aggregation operators.

4.2. Discussion and analysis

The comparative analysis of the HFWA, HFWG, and proposed HFW-AG operators highlights their distinct aggregation behaviors under hesitant fuzzy information. The HFWA operator generally yields higher aggregation values than the HFWG operator because of its additive nature, while the HFWG operator produces comparatively lower values because of its multiplicative structure. The proposed HFW-AG operator produces distinct values by integrating both additive and multiplicative characteristics to provide a balanced representation of the input information. This behavior leads to a stable ranking of alternatives by reducing the bias associated with purely additive and purely multiplicative aggregation strategies, while also providing discrimination among alternatives through moderation of extreme effects. Furthermore, the hybrid formulation of the HFW-AG operator enhances flexibility in modeling DMs' preferences by simultaneously capturing arithmetic and geometric perspectives within hesitant fuzzy environments. Overall, the proposed operator serves as a robust aggregation tool and provides a meaningful contribution to existing hesitant fuzzy aggregation approaches.

5. Conclusion

In this study, the HFS aggregation technique and its application to MCGDM problems were investigated. A new HFW-AG operator was proposed to aggregate hesitant fuzzy information in uncertain decision-making environments. The proposed operator combines characteristics of the HFWA and HFWG operators, offering an alternative approach for aggregating hesitant fuzzy data. Several mathematical properties of the HFW-AG operator were examined to verify its validity. In addition, a numerical example concerning the selection of an air-conditioning system for a university auditorium hall was presented to illustrate the applicability of the proposed method. The comparative results obtained in the considered example indicate that the HFW-AG operator provides competitive aggregation outcomes relative to the HFWA and HFWG operators. These findings suggest that the proposed approach may be useful for handling uncertainty and hesitation in MCGDM problems and may assist decision-makers during the evaluation process. Therefore, this study contributes to the development of aggregation techniques in hesitant fuzzy decision-making and provides a foundation for future research. Future studies may focus on developing generalized forms of the proposed operator and extending its use to other fuzzy environments, such as triangular fuzzy sets, Pythagorean hesitant fuzzy sets, and neutrosophic sets.

Data availability

The data supporting the findings of this study are included within the article. No additional datasets were generated or analyzed during the current study.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Funding

The authors received no external funding for this study.

Acknowledgment

The authors would like to express their sincere gratitude to the Editor and the anonymous reviewers for their valuable comments and constructive suggestions, which significantly improved the quality and presentation of this manuscript.

References

[1] M. Harvey, R. Fisher, R. McPhail & M. Moeller, "Globalization and its impact on global managers' decision processes", Human Resource Development International **12** (2009) 353. https://doi.org/10.1080/13678860903135730.
[2] T. Blumentritt, "The big picture: decision making and globalization", Journal of Emerging Knowledge on Emerging Markets **3** (2011) 426. https://doi.org/10.7885/1946-651X.1057.
[3] Y. Rong, W. Niu, H. Garg, Y. Liu & L. Yu, "A hybrid group decision approach based on MARCOS and regret theory for pharmaceutical enterprises assessment under a single-valued neutrosophic scenario", Systems **10** (2022) 106. https://doi.org/10.3390/systems10040106.
[4] M. Akram, W. A. Dudek & F. Ilyas, "Group decision-making based on Pythagorean fuzzy TOPSIS method", International Journal of Intelligent Systems **34** (2019) 1455. https://doi.org/10.1002/int.22103.
[5] Y. Wu, Y. Gao, B. Zhang & W. Pedrycz, "Minimum information-loss transformations to support heterogeneous group decision making in a distributed linguistic context", Information Fusion **89** (2023) 437. https://doi.org/10.1016/j.inffus.2022.07.009.
[6] L. D. D. R. Calache, V. C. B. Camargo, L. Osiro & L. C. R. Carpinetti, "A genetic algorithm based on dual hesitant fuzzy preference relations for consensus group decision making", Applied Soft Computing **121** (2022) 108778. https://doi.org/10.1016/j.asoc.2022.108778.
[7] I. M. Sharaf, "The differential measure for Pythagorean fuzzy multiple criteria group decision-making", Complex & Intelligent Systems **9** (2023) 3333. https://doi.org/10.1007/s40747-022-00913-4.
[8] L. A. Zadeh, "Fuzzy sets", Information and Control **8** (1965) 338. https://doi.org/10.1016/S0019-9958(65)90241-X.
[9] K. T. Atanassov & S. Stoeva, "Intuitionistic fuzzy sets", Fuzzy Sets and Systems **20** (1986) 87. https://doi.org/10.1016/S0165-0114(86)80034-3.
[10] D. J. Dubois, *Fuzzy Sets and Systems: Theory and Applications*, 1st ed., Academic Press, New York, NY, USA, 1980, pp. 149–334. Available online: https://shop.elsevier.com/books/fuzzy-sets-and-systems/dubois/978-0-12-222750-9.

[11] S. Miyamoto, "Remarks on basics of fuzzy sets and fuzzy multisets", Fuzzy Sets and Systems **156** (2005) 427. https://doi.org/10.1016/j.fss.2005.05.040.
[12] R. R. Yager, "On the theory of bags", International Journal of General Systems **13** (1986) 23. https://doi.org/10.1080/03081078608934952.
[13] S. Miyamoto, *Multisets and fuzzy multisets*, in *Soft Computing and Human-Centered Machines*, Springer, Tokyo, Japan, 2000, pp. 9–33. https://doi.org/10.1007/978-4-431-67907-3_2.
[14] V. Torra, "Hesitant fuzzy sets", International Journal of Intelligent Systems **25** (2010) 529. https://doi.org/10.1002/int.20418.
[15] V. Torra & Y. Narukawa, "On hesitant fuzzy sets and decision", 2009 IEEE International Conference on Fuzzy Systems, Jeju Island, Korea, 2009, pp. 1378–1382. https://doi.org/10.1109/FUZZY.2009.5276884.
[16] M. Xia & Z. Xu, "Hesitant fuzzy information aggregation in decision making", International Journal of Approximate Reasoning **52** (2011) 395. https://doi.org/10.1016/j.ijar.2010.09.002.
[17] U. F. Simo & H. Gwét, "Fuzzy triangular aggregation operators", International Journal of Mathematics and Mathematical Sciences **2018** (2018) 9209524. https://doi.org/10.1155/2018/9209524.
[18] J. L. Marichal, Aggregation Operators for Multicriteria Decision Aid, Ph.D. dissertation, University of Liège, Liège, Belgium, 1998. Available online: https://hdl.handle.net/10993/7224.
[19] J. Ye, "Similarity measures between interval neutrosophic sets and their applications in multicriteria decision-making", Journal of Intelligent & Fuzzy Systems **26** (2014) 165. https://doi.org/10.3233/IFS-120724.
[20] S. Broumi & F. Smarandache, "Several similarity measures of neutrosophic sets", Neutrosophic Sets and Systems **1** (2013) 54. Available online: https://digitalrepository.unm.edu/nss_journal/vol1/iss1/10.
[21] N. Chen, Z. Xu & M. Xia, "Correlation coefficients of hesitant fuzzy sets and their applications to clustering analysis", Applied Mathematical Modelling **37** (2013) 2197. https://doi.org/10.1016/j.apm.2012.04.031.
[22] J. Ye, "Multi-criteria decision-making method using the correlation coefficient under single-valued neutrosophic environment", International Journal of General Systems **42** (2013) 386. https://doi.org/10.1080/03081079.2012.761609.
[23] G. Şenel, J.-G. Lee & K. Hur, "Distance and similarity measures for octahedron sets and their application to MCGDM problems", Mathematics **8** (2020) 1690. https://doi.org/10.3390/math8101690.
[24] J. Lee, G. Şenel, K. Hur & J.-G. Lee, "Octahedron sets", Annals of Fuzzy Mathematics and Informatics **19** (2020) 211. https://doi.org/10.30948/afmi.2020.19.3.211.
[25] Z. Xu & Q. Chen, "A multicriteria decision making procedure based on interval-valued intuitionistic fuzzy Bonferroni means", Journal of Systems Science and Systems Engineering **20** (2011) 217. https://doi.org/10.1007/s11518-011-5163-0.
[26] P. Liu, S.-M. Chen & J. Liu, "Multiple attribute group decision making based on intuitionistic fuzzy interaction partitioned Bonferroni mean operators", Information Sciences **411** (2017) 98. https://doi.org/10.1016/j.ins.2017.05.016.
[27] H. Garg & R. Arora, "Bonferroni mean aggregation operators under intuitionistic fuzzy soft set environment and their applications to decision-making", Journal of the Operational Research Society **69** (2018) 1711. https://doi.org/10.1080/01605682.2017.1409159.
[28] G. Kaur & H. Garg, "Multi-attribute decision-making based on Bonferroni mean operators under cubic intuitionistic fuzzy set environment", Entropy **20** (2018) 65. https://doi.org/10.3390/e20010065.
[29] Z. Zhang, "Interval-valued intuitionistic hesitant fuzzy aggregation operators and their application in group decision-making", Journal of Applied Mathematics **2013** (2013) 670285. https://doi.org/10.1155/2013/670285.
[30] H. Liao & Z. Xu, *Hesitant fuzzy set and its extensions*, in *Hesitant Fuzzy Decision Making Methodologies and Applications*, Springer, Singapore, 2017, pp. 1–36. https://doi.org/10.1007/978-981-10-3265-3_1.
[31] Z. Xu, "On consistency of the weighted geometric mean complex judgment matrix in AHP", European Journal of Operational Research **126** (2000) 683. https://doi.org/10.1016/S0377-2217(99)00082-X.
[32] V. Torra & Y. Narukawa, *Modeling Decisions: Information Fusion and Aggregation Operators*, Springer, Berlin, Germany, 2007. https://doi.org/10.1007/978-3-540-68791-7.
[33] M. Sarwar, G. Ali, W. Gulzar, M. Akram & D. Pamucar, "Two-sided matching optimization model for green housing technology selection based on hesitant 2-tuple linguistic rough numbers", Engineering Applications of Artificial Intelligence **151** (2025) 110559. https://doi.org/10.1016/j.engappai.2025.110559.
[34] M. Akram, X. Peng & A. Sattar, "Multi-criteria decision-making model using complex Pythagorean fuzzy Yager aggregation operators", Arabian Journal for Science and Engineering **46** (2021) 1691. https://doi.org/10.1007/s13369-020-04864-1.
[35] A. Sarkar, S. Moslem, D. Esztergár-Kiss, M. Akram, L. Jin & T. Senapati, "A hybrid approach based on dual hesitant q-rung orthopair fuzzy Frank power partitioned Heronian mean aggregation operators for estimating sustainable urban transport solutions", Engineering Applications of Artificial Intelligence **124** (2023) 106505. https://doi.org/10.1016/j.engappai.2023.106505.
[36] M. Akram, X. Peng & A. Sattar, "A new decision-making model using complex intuitionistic fuzzy Hamacher aggregation operators", Soft Computing **25** (2021) 7059. https://doi.org/10.1007/s00500-021-05658-9.
[37] B. Ning, Y. Zhang, C. Wei & G. Wei, "Some generalized aggregation operators with probabilistic spherical hesitant fuzzy information and applications to green enterprise credit selection", Informatica **37** (2026) 435. https://doi.org/10.15388/26-INFOR623.
[38] M. Kirişci, S. Kuzu & A. Kablan, "The investment portfolio selection with a hesitant fuzzy decision-making method based on aggregation operators", Borsa Istanbul Review **26** (2026) 100805. https://doi.org/10.1016/j.bir.2026.100805.
[39] R. Ma & W. Zeng, "Bonferroni mean operators on weighted hesitant fuzzy set and its application in multi-criteria symmetric group decision making", Symmetry **18** (2026) 414. https://doi.org/10.3390/sym18030414.