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An improved exponential ratio estimator for post-stratified sampling designs

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Abstract

This study proposes a new ratio-type exponential estimator for estimating the finite population mean under post-stratification. A simple random sample is first selected and later divided into post-strata using an auxiliary variable to improve the precision of estimation. The proposed estimator incorporates auxiliary information obtained through post-stratification to reduce the mean squared error (MSE) compared with the usual unbiased estimator and some existing ratio-type estimators. Expressions for the bias and MSE of the estimator are derived to the first order of approximation, and the conditions under which the estimator performs more efficiently are established. To examine its practical usefulness, a real population dataset was employed and analysed. The performance of the proposed estimator was assessed using percentage relative efficiency (PRE) measures and compared with those of existing estimators available in the literature. The empirical results indicate that the proposed estimator produces smaller MSE values and higher efficiency values than the competing estimators. The findings therefore show that the estimator provides a more reliable and efficient approach for estimating the population mean, particularly in surveys involving stratified population data and the availability of relevant auxiliary information.

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1. Introduction

The use of post-stratification and auxiliary information in survey sampling has been thoroughly investigated to enhance the precision of population mean estimates. Various estimation procedures, such as ratio and product estimators, have been developed to reduce estimation errors. Initial research focused on applying post-stratification, using auxiliary information, to improve the precision and accuracy of population mean estimates in surveys [1]. Ratio- and product-type exponential estimators were subsequently

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designed to improve estimation accuracy [2, 3]. More recent research has extended this area through the development of improved ratio- and product-type exponential estimators for finite populations using post-stratification [4, 5]. Along similar lines, a novel ratio-type estimator was introduced to estimate the population mean with the aid of post-stratification [4, 5]. Collectively, the cited studies show efforts by researchers to contribute to improved estimation techniques, particularly methods for enhancing the efficiency of post-stratification.

Although considerable research has been conducted on exponential ratio estimators using stratified and simple random sampling techniques, less attention has been given to their performance in post-stratified sample designs. Post-stratification introduces additional structure and variability to survey samples, which may influence the performance of exponential ratio estimators differently from conventional sampling approaches. Understanding these effects is significant for advancing both the theoretical foundation and practical application of survey methodology.

The aim of this work is to examine the performance of exponential ratio estimators in post-stratified sample designs, particularly their bias and mean squared error (MSE) properties. The research provides theoretical guidelines and empirical evidence for situations in which these estimators are more efficient than traditional estimators. This study is in line with ongoing efforts to improve estimation techniques for post-stratification in order to increase efficiency and accuracy.

2. Methodology

This study considered a finite population of size N divided into L post-strata such that the hth stratum contains N_h units ($h = 1, 2, \ldots, L$) with

$$\sum_{h=1}^{L} N_h = N. \tag{1}$$

A sample size n is drawn using simple random sampling without replacement (SRSWOR), and the units are classified into L post-strata such that

$$\sum_{h=1}^{L} n_h = n. \tag{2}$$

The unbiased estimator for the population mean is presented as

$$\overline{y}_{PS} = \sum_{h=1}^{L} W_h \overline{y}_h, \tag{3}$$

where $W_h = N_h/N$ is the weight of the hth stratum and $\overline{y}_h = (1/n_h)\sum_{i=1}^{n_h} y_{hi}$ is the sample mean of n_h sample units in the hth stratum. The corresponding variance of $\overline{y}_{PS}$ is obtained using the results from Ref. [5]:

$$\operatorname{Var}(\overline{y}_{PS}) = \lambda \sum_{h=1}^{L} W_h S_{yh}^2 + \frac{1}{n^2} \sum_{h=1}^{L} (1 - W_h) S_{yh}^2, \tag{4}$$

where

$$S_{yh}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} \left(y_{hi} - \overline{Y}_h\right)^2. \tag{5}$$

A product-type estimator for post-stratification was suggested by Ref. [1] as

$$\widehat{\overline{Y}}_{PS}^{t} = \overline{y}_{PS} \left(\frac{\overline{x}_{PS}}{\overline{X}}\right), \tag{6}$$

where $\overline{X} = \sum_{h=1}^{L} W_h \overline{X}_h$, $\overline{y}_{PS} = \sum_{h=1}^{L} W_h \overline{y}_h$, and $\overline{x}_{PS} = \sum_{h=1}^{L} W_h \overline{x}_h$. The bias and MSE of $\widehat{\overline{Y}}_{PS}^{t}$ to the first order of approximation are defined as

$$B\left(\widehat{\overline{Y}}_{PS}^{t}\right) = \lambda \frac{1}{\overline{X}} \sum_{h=1}^{L} W_h \left(R_1 S_{xh}^2 + S_{yxh}\right), \tag{7}$$

and

$$\operatorname{MSE}\left(\widehat{\overline{Y}}_{PS}^{t}\right) = \lambda \sum_{h=1}^{L} W_h \left(S_{yh}^2 + R_1^2 S_{xh}^2 + 2R_1 S_{yxh}\right), \tag{8}$$

where $R_1 = \overline{Y}/\overline{X}$.

The classical ratio-type estimator in post-stratification was defined by Refs. [1, 4, 6] as

$$\widehat{\overline{Y}}_{PS}^{R} = \overline{y}_{PS}\left(\frac{\overline{X}}{\overline{x}_{PS}}\right). \tag{9}$$

The bias and MSE of $\widehat{\overline{Y}}_{PS}^{R}$ up to the first order of approximation are defined as

$$B\left(\widehat{\overline{Y}}_{PS}^{R}\right) = \lambda\frac{1}{\overline{X}}\sum_{h=1}^{L} W_h\left(R_1 S_{xh}^2 - S_{yxh}\right), \tag{10}$$

and

$$\mathrm{MSE}\left(\widehat{\overline{Y}}_{PS}^{R}\right) = \lambda\sum_{h=1}^{L} W_h\left(S_{yh}^2 + R_1^2 S_{xh}^2 - 2R_1 S_{yxh}\right), \tag{11}$$

where $R_1 = \overline{Y}/\overline{X}$.

Ref. [4] defined the ratio exponential estimator as

$$\widehat{\overline{Y}}_{PS}^{RE} = \overline{y}_{PS}\exp\left(\frac{\overline{X} - \overline{x}_{PS}}{\overline{X} + \overline{x}_{PS}}\right). \tag{12}$$

The bias and MSE of $\widehat{\overline{Y}}_{PS}^{RE}$ up to the first order of approximation are respectively given by

$$B\left(\widehat{\overline{Y}}_{PS}^{RE}\right) = \lambda\frac{1}{\overline{X}}\sum_{h=1}^{L} W_h\left(\frac{3}{8}R_1 S_{xh}^2 - \frac{1}{2}S_{yxh}\right), \tag{13}$$

and

$$\mathrm{MSE}\left(\widehat{\overline{Y}}_{PS}^{RE}\right) = \lambda\sum_{h=1}^{L} W_h\left(S_{yh}^2 + \frac{1}{4}R_1^2 S_{xh}^2 - R_1 S_{yxh}\right). \tag{14}$$

2.1. Proposed estimator

Ref. [6] explored the ratio estimator in stratified sampling using information on two auxiliary variables. The study variable and the two auxiliary variables have the same significance, and the estimator is presented as

$$\widehat{\overline{Y}}_{RR}^{st} = \overline{y}_{st}\left(\frac{\overline{X}}{\overline{x}_{st}}\right)^{\delta}\left(\frac{\overline{Z}}{\overline{z}_{st}}\right). \tag{15}$$

The work of Ref. [7] prompted the development of a ratio and ratio exponential estimator for the population mean as

$$\begin{aligned}\widehat{\overline{Y}}_{S} &= \overline{y}_{PS}\exp\left[\frac{\sum_{h=1}^{L} W_h\overline{X}_h - \sum_{h=1}^{L} W_h\overline{x}_h}{\sum_{h=1}^{L} W_h\overline{X}_h + \sum_{h=1}^{L} W_h\overline{x}_h}\right]\\ &\quad\times\exp\left[\frac{\sum_{h=1}^{L} W_h\overline{z}_h - \sum_{h=1}^{L} W_h\overline{Z}_h}{\sum_{h=1}^{L} W_h\overline{z}_h + \sum_{h=1}^{L} W_h\overline{Z}_h}\right].\end{aligned} \tag{16}$$

$\widehat{\overline{Y}}_{S}$ can also be expressed as

$$\widehat{\overline{Y}}_{S} = \overline{y}_{PS}\exp\left(\frac{\overline{X} - \overline{x}_{PS}}{\overline{X} + \overline{x}_{PS}}\right)\exp\left(\frac{\overline{z}_{PS} - \overline{Z}}{\overline{z}_{PS} + \overline{Z}}\right). \tag{17}$$

We write

$$\overline{y}_h = \overline{Y}_h(1+\varepsilon_{0h}), \quad \overline{x}_h = \overline{X}_h(1+\varepsilon_{1h}), \quad \overline{z}_h = \overline{Z}_h(1+\varepsilon_{2h}), \tag{18}$$

such that

$$E(\varepsilon_{0h}) = E(\varepsilon_{1h}) = E(\varepsilon_{2h}) = 0, \tag{19}$$

$$\begin{aligned}E(\varepsilon_{0h}^2) &= \left(\frac{1}{nW_h} - \frac{1}{N_h}\right)C_{yh}^2,\\ E(\varepsilon_{1h}^2) &= \left(\frac{1}{nW_h} - \frac{1}{N_h}\right)C_{xh}^2,\\ E(\varepsilon_{2h}^2) &= \left(\frac{1}{nW_h} - \frac{1}{N_h}\right)C_{zh}^2,\end{aligned} \tag{20}$$

and

$$\begin{aligned} E(\varepsilon_{0h}\varepsilon_{1h}) &= \left(\frac{1}{nW_h} - \frac{1}{N_h}\right)\rho_{yxh}C_{yh}C_{xh}, \\ E(\varepsilon_{0h}\varepsilon_{2h}) &= \left(\frac{1}{nW_h} - \frac{1}{N_h}\right)\rho_{yzh}C_{yh}C_{zh}, \\ E(\varepsilon_{1h}\varepsilon_{2h}) &= \left(\frac{1}{nW_h} - \frac{1}{N_h}\right)\rho_{xzh}C_{xh}C_{zh}. \end{aligned} \tag{21}$$

The suggested estimator $\widehat{\overline{Y}}_S$ can be presented in terms of the ε_i's as

$$\widehat{\overline{Y}}_S = \overline{Y}(1+\varepsilon_0)\left(\frac{1}{1+\varepsilon_1}\right)^{\delta}\left(\frac{1}{1+\varepsilon_2}\right). \tag{22}$$

Keeping terms up to the second order gives

$$\widehat{\overline{Y}}_S = \overline{Y}\Big[1 + \varepsilon_0 - \delta\varepsilon_1 - \varepsilon_2 + \left(\frac{\delta}{2} + \frac{\delta^2}{2}\right)\varepsilon_1^2 + \varepsilon_2^2 + \delta\varepsilon_1\varepsilon_2 - \delta\varepsilon_0\varepsilon_1 - \varepsilon_0\varepsilon_2\Big]. \tag{23}$$

where

$$\varepsilon_0 = \frac{\sum_{h=1}^{L} W_h\overline{Y}_h\varepsilon_{0h}}{\overline{Y}}, \quad \varepsilon_1 = \frac{\sum_{h=1}^{L} W_h\overline{X}_h\varepsilon_{1h}}{\overline{X}}, \quad \varepsilon_2 = \frac{\sum_{h=1}^{L} W_h\overline{Z}_h\varepsilon_{2h}}{\overline{Z}}, \tag{24}$$

such that $E(\varepsilon_0) = E(\varepsilon_1) = E(\varepsilon_2) = 0$ and

$$\begin{aligned} E(\varepsilon_0^2) &= \frac{\lambda}{\overline{Y}^2}\sum_{h=1}^{L} W_h S_{yh}^2, \\ E(\varepsilon_1^2) &= \frac{\lambda}{\overline{X}^2}\sum_{h=1}^{L} W_h S_{xh}^2, \\ E(\varepsilon_2^2) &= \frac{\lambda}{\overline{Z}^2}\sum_{h=1}^{L} W_h S_{zh}^2, \end{aligned} \tag{25}$$

$$\begin{aligned} E(\varepsilon_0\varepsilon_1) &= \frac{\lambda}{\overline{Y}\,\overline{X}}\sum_{h=1}^{L} W_h S_{yxh}, \\ E(\varepsilon_0\varepsilon_2) &= \frac{\lambda}{\overline{Y}\,\overline{Z}}\sum_{h=1}^{L} W_h S_{yzh}, \\ E(\varepsilon_1\varepsilon_2) &= \frac{\lambda}{\overline{X}\,\overline{Z}}\sum_{h=1}^{L} W_h S_{xzh}. \end{aligned} \tag{26}$$

The bias and MSE of the suggested estimator are obtained up to the first order of approximation as

$$\begin{aligned} \text{Bias}(\widehat{\overline{Y}}_S) = \overline{Y}\lambda\Big[&\left(\frac{\delta+\delta^2}{2\overline{X}^2}\right)\sum_{h=1}^{L} W_h S_{xh}^2 + \frac{1}{\overline{Z}^2}\sum_{h=1}^{L} W_h S_{zh}^2 \\ &+ \frac{\delta}{\overline{XZ}}\sum_{h=1}^{L} W_h S_{xzh} - \frac{\delta}{\overline{YX}}\sum_{h=1}^{L} W_h S_{yxh} - \frac{1}{\overline{YZ}}\sum_{h=1}^{L} W_h S_{yzh}\Big], \end{aligned} \tag{27}$$

and

$$\begin{aligned} \text{MSE}(\widehat{\overline{Y}}_S) = \lambda\sum_{h=1}^{L} W_h S_{yh}^2 + \overline{Y}^2\lambda\Big[&\frac{\delta^2}{\overline{X}^2}\sum_{h=1}^{L} W_h S_{xh}^2 + \frac{1}{\overline{Z}^2}\sum_{h=1}^{L} W_h S_{zh}^2 \\ &- \frac{2\delta}{\overline{YX}}\sum_{h=1}^{L} W_h S_{yxh} - \frac{2}{\overline{YZ}}\sum_{h=1}^{L} W_h S_{yzh} + \frac{2\delta}{\overline{XZ}}\sum_{h=1}^{L} W_h S_{xzh}\Big]. \end{aligned} \tag{28}$$

Table 1: Population constants used for the empirical analysis.

Population constant	Stratum I	Stratum II
Population size, N_h	5	5
Sample size, n_h	4	4
Mean output, $\bar{Y}_h$	1925.00	3115.60
Mean fixed capital, $\bar{X}_h$	214.40	333.80
Mean number of workers, $\bar{Z}_h$	51.80	60.60
Standard deviation of output, S_{yh}	615.92	340.38
Standard deviation of fixed capital, S_{xh}	74.87	66.35
Standard deviation of number of workers, S_{zh}	0.75	4.84
Covariance between output and fixed capital, S_{yxh}	39,360.68	22,356.50
Covariance between output and number of workers, S_{yzh}	−411.20	−1, 536.00
Covariance between fixed capital and number of workers, S_{xzh}	38.08	287.92

3. Results

3.1. Theoretical efficiency comparison

Comparison of equations (4), (8), (11), and (14) shows that the proposed estimator $\widehat{\overline{\overline{Y}}}_S$ would be more efficient if

(i) $\mathrm{Var}(\bar{y}_{PS}) - \mathrm{MSE}(\widehat{\overline{\overline{Y}}}_S) > 0$:

$$\frac{1}{n^2}\sum_{h=1}^{L}(1-W_h)S_{yh}^2 - \alpha > 0. \tag{29}$$

(ii) $\mathrm{MSE}(\widehat{\overline{\overline{Y}}}{}^{t}_{PS}) - \mathrm{MSE}(\widehat{\overline{\overline{Y}}}_S) > 0$:

$$R_1^2 S_{xh}^2 + 2R_1 S_{yxh} - \alpha > 0. \tag{30}$$

(iii) $\mathrm{MSE}(\widehat{\overline{\overline{Y}}}{}^{R}_{PS}) - \mathrm{MSE}(\widehat{\overline{\overline{Y}}}_S) > 0$:

$$R_1^2 S_{xh}^2 - 2R_1 S_{yxh} - \alpha > 0. \tag{31}$$

(iv) $\mathrm{MSE}(\widehat{\overline{\overline{Y}}}{}^{RE}_{PS}) - \mathrm{MSE}(\widehat{\overline{\overline{Y}}}_S) > 0$:

$$\frac{1}{4}R_1^2 S_{xh}^2 - R_1 S_{yxh} - \alpha > 0, \tag{32}$$

where

$$\alpha = \left[\frac{R_1^2 S_{xh}^2}{4} + \frac{R_2^2 S_{zh}^2}{4} - R_1 S_{yxh} + R_2 S_{yzh} + \frac{1}{2}R_1 R_2 S_{xzh}\right]. \tag{33}$$

3.2. Empirical study

To compare the proposed estimator with the existing estimators, a dataset was considered from Ref. [8]. In the dataset, y denotes output, x denotes fixed capital, and z denotes number of workers.

For the empirical analysis, a real-world dataset sourced from Murthy (Ref. [8]) was employed. The dataset comprises information on output, fixed capital, and number of workers, categorized into two post-strata. Using these data, the estimators were applied to evaluate their performance in terms of MSE, PRE, and relative bias. The objective was to assess the efficiency and accuracy of the proposed estimator compared with existing estimators. Table 1 shows the population constants used for the empirical analysis.

To evaluate the performance of the proposed estimator, a comparative analysis was conducted using both theoretical and empirical metrics. The estimators were evaluated using their MSE, PRE, and relative bias. The results indicate how the proposed estimator improves performance and reduces errors compared with the traditional and other existing estimators.

The results presented in Tables 2–5 indicate that the proposed ratio-type exponential estimator performs better than the other estimators under post-stratification. Table 2 shows that the proposed estimator has the lowest MSE, indicating that it provides more precise results than the classical estimator and the other existing estimators. This is supported by Table 3, which shows that the proposed estimator has the highest PRE of 316.12%, meaning that it performs better than the unbiased estimator and the classical ratio estimator. The relative-bias results presented in Table 4 show that all the estimators have relatively small biases. The unbiased post-stratified estimator has, by definition, a relative bias of zero. Among the biased estimators, the existing exponential estimator records the smallest relative bias of 0.0066%, followed by the classical ratio estimator with 0.0548%. The proposed estimator has

Table 2: Mean squared errors (MSEs) of different estimators.

Estimator	MSE
Unbiased estimator	10059.07
Classical ratio estimator	9867.27
Existing exponential estimator	3558.77
Proposed estimator	3182.05

Lower MSE values indicate higher precision of the estimator.

Table 3: Percentage relative efficiency (PRE) compared with the unbiased estimator.

Estimator	PRE (%)
Unbiased estimator	100.00
Classical ratio estimator	101.94
Existing exponential estimator	282.66
Proposed estimator	316.12

PRE indicates the gain in efficiency over the unbiased estimator.

Table 4: Derived relative bias of the estimators.

Estimator	Estimated bias	Relative bias (%)
Unbiased estimator, $\bar{y}_{PS}$	0.0000	0.0000
Classical ratio estimator, $\hat{\bar{Y}}_{PS}^{R}$	1.3819	0.0548
Existing exponential estimator, $\hat{\bar{Y}}_{PS}^{RE}$	0.1664	0.0066
Proposed estimator, $\hat{\bar{Y}}_{S}$	2.7210	0.1080

Table 5: Percentage improvement in efficiency over the classical ratio estimator.

Estimator	Improvement (%)
Existing exponential estimator	177.27
Proposed estimator	210.09

a relative bias of 0.1080%. Although the proposed estimator does not produce the smallest relative bias, its bias remains negligible and is accompanied by the lowest mean squared error and highest percentage relative efficiency. Its superior overall performance therefore arises mainly from a substantial reduction in variance rather than from having the smallest bias. Table 5 also indicates that the proposed estimator is more efficient than the classical ratio estimator, as the efficiency gain is over 210%. The findings illustrated in the tables show that the proposed estimator is effective and has better practical characteristics for improving the estimation of population means by using post-stratification.

4. Conclusion

The current study developed and assessed the performance of a new ratio-type exponential estimator for estimating the population mean using the post-stratification approach. Theoretical derivations and empirical analysis show that the new estimator performs significantly better than the existing estimators in terms of precision and accuracy. This is shown by the lowest MSE and highest PRE, thereby corroborating its statistical superiority. This increase in efficiency is particularly important when dealing with stratified population data because auxiliary information should be used optimally. Overall, this approach is a viable tool for researchers seeking to improve the reliability of population mean estimates in post-stratified surveys.

Data availability

Data will be made available upon reasonable request from the corresponding author.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this manuscript.

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References

[1] F. A. Ige & T. P. Tripathi, "Estimation of population mean using post-stratification and auxiliary information", Abacus **18** (1989) 265.
[2] R. Tailor, R. Tailor & S. Chouhan, "Improved ratio- and product-type exponential estimators for population mean in case of post-stratification", Communications in Statistics – Theory and Methods **46** (2017) 10387. https://doi.org/10.1080/03610926.2012.732180.
[3] S. Bahl & R. K. Tuteja, "Ratio and product type exponential estimators", Journal of Information and Optimization Sciences **12** (1991) 159. https://doi.org/10.1080/02522667.1991.10699058.
[4] R. Tailor & A. Mehta, "A ratio and ratio exponential estimator for finite population mean in case of post-stratification", Journal of Statistics Applications & Probability **8** (2019) 241. https://doi.org/10.18576/jsap/080308.
[5] K. U. I. Rather, M. I. Jeelani, M. Y. Shah, S. E. H. Rizvi & M. Sharma, "A new ratio type estimator for computation of population mean under post-stratification", Journal of Applied Mathematics and Statistical Informatics **18** (2022) 29. https://doi.org/10.2478/jamsi-2022-0003.
[6] N. Koyuncu & C. Kadilar, "Ratio and product estimators in stratified random sampling", Journal of Statistical Planning and Inference **139** (2009) 2552. https://doi.org/10.1016/j.jspi.2008.11.009.
[7] W. G. Cochran, "The estimation of yields of cereal experiments by sampling for the ratio of grain to total produce", Journal of Agricultural Science **30** (1940) 262. https://doi.org/10.1017/S0021859600048012.
[8] M. N. Murthy, Sampling Theory and Methods, Statistical Publishing Society, Calcutta, India, 1967.