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Numerical simulation of a nonlinear diffusion type equation in a two phase media with linear porosity and permeability model

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Abstract

Predicting and understanding the behavior of pressure is important in reservoir maintenance and evaluation. This work studies the behavior of fluid pressure in a reservoir by numerical simulation of the pressure diffusion type equation in a two phase media with a linear porosity and permeability model. Because the porosity and permeability are pressure dependent, the resulting diffusion type equation is nonlinear and is solved using a backward-forward finite difference method. The simulation code was ran using a constant porosity and permeability model and a linear porosity and permeability model. The results from the linear porosity and permeability model was compared to that of the constant porosity and permeability model. In both cases the pressure gradient was greatest at the wellbore and decreases as the radial distances away from the wellbore increases. The pressure in both cases also decreased with time. However, at each location and time the pressure drop was lower in the linear porosity and permeability model than with the constant porosity and permeability. The backward-forward finite difference method proved to be useful in solving numerically the nonlinear diffusion type equation. This work can be applied in the oil and gas industry to predict pressure behavior in reservoirs and make investment decisions, production and maintenance decision.

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1. Introduction

The problem of fluid flow in porous media can be better understood if one takes a look at fluid flows in pipes and water channels. It is rather easy to take measurements such as the length and diameter of a pipe and compute its flow capacity and pressure as a function of distance and time [1]. In pipes the volume of fluid delivered at the open end depends on the pressure difference and partly on the cross-sectional area of the pipes if the pressure is high [2]. The flow of water in channels also depends on pressure difference but is often coded in terms of topography of the land. When the pressure in a pipe is very high, the pipe may either burst or the rate of fluid delivery will increase. Under the ground there are no such pipes, no clear-cut flow paths that can be used for measurements. The fine capillary like pores through which the fluid flows are not of any uniform cross-section nor can they be said to be rigid. The consequence is that the flow parameters such as porosity of the media through which fluids flow beneath the earth surface and the

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permeability of the fluid may in fact depend on the fluid pressure [3]. The models of fluid flow incorporating functional dependence of media and fluid properties on pressure results to complex and nonlinearity of the governing equations.

Due to complex nature of porous media (for example, a reservoir) various researchers were attracted to tackle the problems related to this topic and formulated different relations for studying diffusion of fluids in porous media. However, most of the researchers derived their models on the assumption of a constant diffusion coefficient (diffusivity constant) [4]. The diffusivity in practice may not be constant and may depend on temperature, concentration, pressure, amongst others. In some attempts, where nonlinearity is retained in the models, they are indirectly avoided at the solution stage through the process of linearization. This is like abandoning the problem at hand to pursue the ghost of it. Also, due to the complex nature of multiphase flow, nonlinearity of their governing equations and reservoir intricacies, finding analytical solutions to practical fluid flow problems is impossible. Therefore, the only means by which such models can be solved is by using numerical methods such as finite difference or finite element [5].

Among the many factors that may cause the low yield in wells, the fluid pressure variation in the well environment is a critical one. Understanding the nature of the fluid pressure distribution in the well environment as influenced by the porosity of the earth formations and the permeability (as both are also influenced by pressure) is very essential in decision making in the sinking of mineral wells. Sudden changes in fluid pressure when drilling a well and well stability can cause major problems when one is unprepared [6]. Thus, pressure prediction can help us to reduce drilling risk and to increase wellbore stability. The solutions of the diffusion type equation with pressure dependent porosity and permeability are very sparse in literature, so the findings of this research may add to the knowledge of nonlinearity in Physics and its applications in oil and gas exploration industry.

2. Materials and Methods

2.1. Governing Laws and Equations

One of the basic laws governing fluid flow through a porous media is the mass conservation law [7],

$$-\frac{\partial(\rho q r)}{\partial r} = r\frac{\partial(\rho \emptyset)}{\partial t}, \tag{1}$$

where ρ = density of the fluid, q = Darcy's velocity, $\emptyset$ = porosity, r = radial distance, t = time taken.

For isothermal fluid flow the mass conservation law can be written as [8];

$$r\frac{\partial(\rho\emptyset)}{\partial t} = r\left[\rho\emptyset\left(c_\emptyset + c_f\right)\frac{\partial p}{\partial t}\right], \tag{2}$$

where

$$c_\emptyset = \frac{1}{\emptyset}\left(\frac{\partial \emptyset}{\partial p}\right)_t. \tag{3}$$

$c_\emptyset$ = isothermal compressibility of porosity

while

$$c_f = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial p}\right)_t. \tag{4}$$

c_f = isothermal compressibility of fluid

$$c_\emptyset + c_f = c_t. \tag{5}$$

c_t = total compressibility

Another basic law governing fluid flow is the Darcy Equation,

$$q = -\frac{k}{\mu}\frac{dp}{dr}, \tag{6}$$

where p = pressure of fluid, k = rock permeability, μ = dynamic fluid viscosity.

Substituting equation (5) and (6) into (2) results to;

$$\frac{1}{r}\frac{\partial}{\partial r}\left[\frac{k}{\mu}\rho r\frac{\partial p}{\partial r}\right] = \rho\emptyset c_t\frac{\partial p}{\partial t}. \tag{7}$$

This is the governing equation for transient radial flow of a fluid through a porous rock [9]. Consider a two phase flow where the fluids are immiscible and there is no mass transfer between the phases. One phase (for example, oil) wets the porous medium more than the other (for example, gas) and is called the wetting phase and indicated by a subscripts, o. The other phase is termed the non-wetting phase and is indicated by g. In general, water is the wetting fluid relative to oil and gas, while oil is the wetting phase relative to gas. Several new quantities peculiar to multiphase flow, such as saturation, capillary pressure and relative permeability

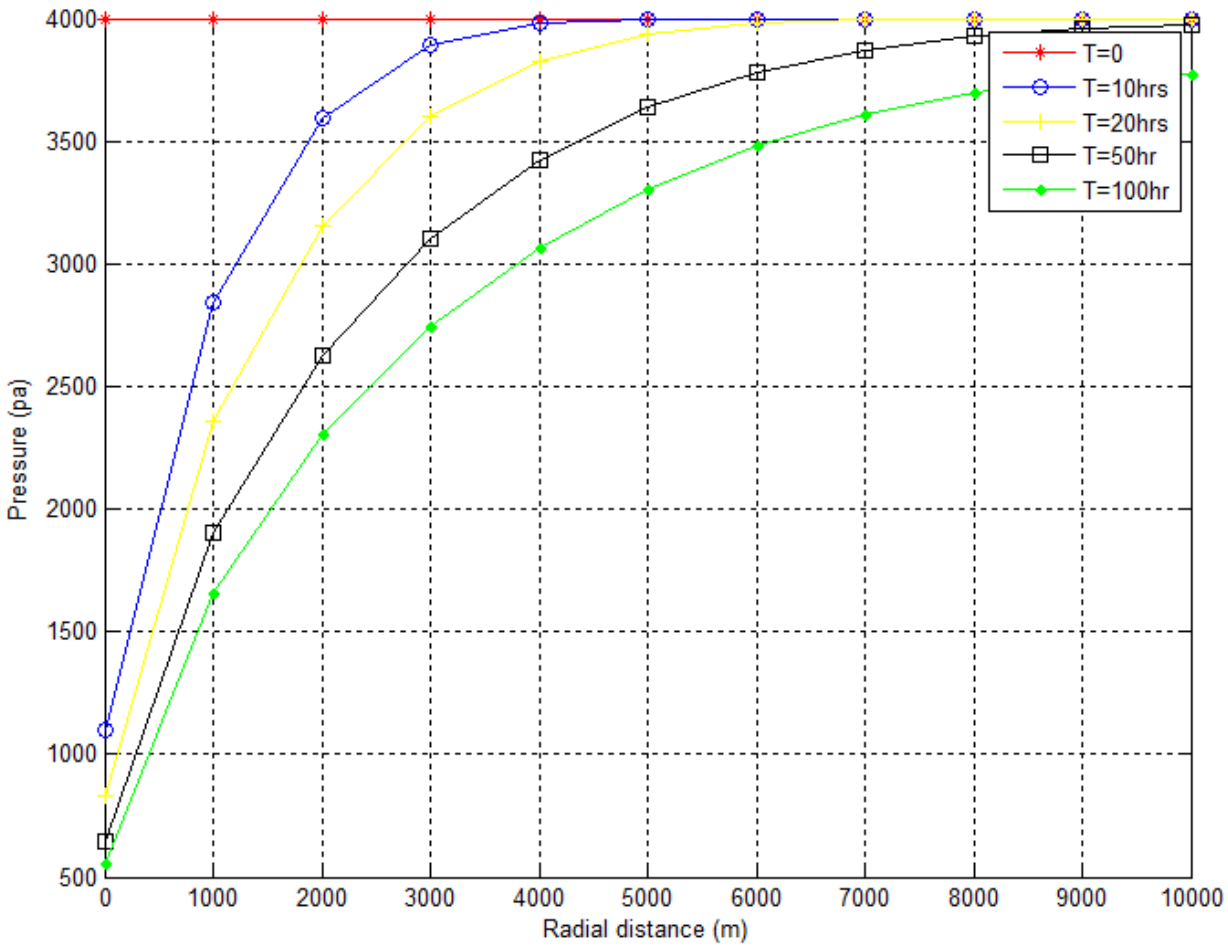

Figure 1: Pressure Distribution for the Constant Porosity and Permeability Model.

must be introduced [10]. The saturation of a fluid phase is defined as the fraction of the void volume of a porous medium filled by the phase [11]. The fact that the two fluids jointly fill the voids implies the relation

$$s_o + s_g = 1, \tag{8}$$

where s_0 and s_g are the saturations of the wetting and non- wetting phases respectively. Also due to the curvature and surface tension of the interface between the two phases, the pressure difference is given by the capillary pressures;

$$p_c = p_g + p_o. \tag{9}$$

Empirically, the capillary pressure is a function of saturation s_0 . Except for the accumulation term, the same derivation that led to equation (6) applies to the mass conservation equation for each fluid phase. Taking into account that there is no mass transfer between phases in the immiscible flow, mass is conserved within each phase and each phase has its own density, ρ and Darcy's velocity, q. The mass conservation equation for each phase is

$$-\frac{\partial(\rho q_o r)}{\partial r} = r\frac{\partial(\emptyset\rho_o s_o)}{\partial t}. \tag{10}$$

$$-\frac{\partial\left(\rho_g q_g r\right)}{\partial r} = r\frac{\partial\left(\emptyset\rho_g s_g\right)}{\partial t}. \tag{11}$$

Darcy's law can be generalized for the two phase flow by also including a relative permeability factor (k_r) for each phase and

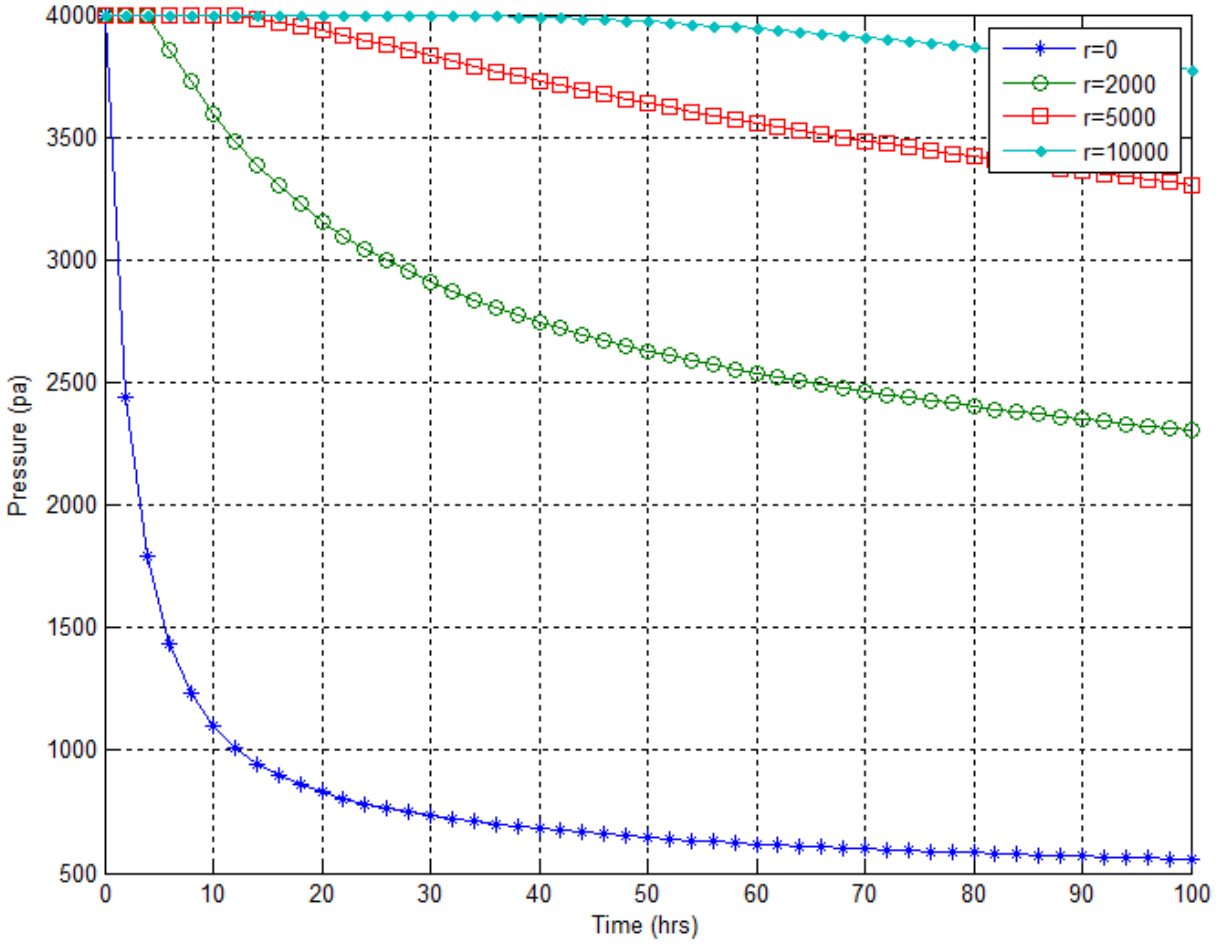

Figure 2: Pressure Profile for the Constant Porosity and Permeability Model.

each phase having its own viscosity and pressure [12, 13].

$$q_o = \frac{-kk_{ro}}{\mu_o}\frac{\partial P_O}{\partial r},\ q_g = \frac{-kk_{rg}}{\mu_g}\frac{\partial p_g}{\partial r}. \tag{12}$$

The two phase flow equation thus reads;

$$\begin{aligned} \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{kk_{ro}}{\mu_o}\rho_o r\frac{\partial p_o}{\partial r}\right) &= \emptyset\rho_o s_o c_t\frac{\partial p_o}{\partial t} \\ \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{kk_{rg}}{\mu_g}\rho_g r\frac{\partial p_g}{\partial r}\right) &= \emptyset\rho_g s_g c_t\frac{\partial \rho_g}{\partial t}. \end{aligned} \tag{13}$$

In the oil and gas reservoirs, the capillary pressure is always much less than p_o or p_g and so we can say $p_o \approx p_g$ and $p_c = 0$. Our equation reduces to a single equation. We will use the equation for the oil phase and replace p_o with p. Equation (13) reduces to;

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{kk_{ro}}{\mu_o}\rho_o r\frac{\partial p}{\partial r}\right) = \emptyset\rho_o s_o c_t\frac{\partial p}{\partial t}. \tag{14}$$

If the fluid properties are kept constant throughout the flow regime such that only the rock properties (porosity and permeability) varies, ignoring the subscripts equation (14) can be simplified as follows;

Bringing out the constants and using product rule to open the brackets,

$$\frac{\rho k_{ro}}{r\mu}\frac{\partial}{\partial r}\left(kr\frac{\partial p}{\partial r}\right) = \emptyset\rho s c_t\frac{\partial p}{\partial t}. \tag{15}$$

$$\frac{k_{ro}}{r\mu}\left[r\frac{\partial k}{\partial r}\frac{\partial p}{\partial r}+k\frac{\partial p}{\partial r}+kr\frac{\partial^2 p}{\partial r^2}\right]=\emptyset sc_t\frac{\partial p}{\partial t}. \tag{16}$$

From the first term on the right in the bracket,

$$r\frac{\partial k}{\partial r}\frac{\partial p}{\partial r}=r\frac{\partial k}{\partial p}\frac{\partial p}{\partial r}\frac{\partial p}{\partial r}$$

$$=r\frac{\partial k}{\partial p}\left(\frac{\partial p}{\partial r}\right)^2. \tag{17}$$

Substituting equation (17) into (16) and simplifying further,

$$\frac{k_{ro}}{\mu}\left[\frac{\partial k}{\partial p}\left(\frac{\partial p}{\partial r}\right)^2+\frac{k}{r}\frac{\partial p}{\partial r}+k\frac{\partial^2 p}{\partial r^2}\right]=\emptyset sc_t\frac{\partial p}{\partial t}. \tag{18}$$

$$\frac{\partial p}{\partial t}=\frac{k_r}{s\mu c_t}\frac{k}{\emptyset}\left[\frac{1}{k}\frac{\partial k}{\partial p}\left(\frac{\partial p}{\partial r}\right)^2+\frac{1}{r}\frac{\partial p}{\partial r}+\frac{\partial^2 p}{\partial r^2}\right]. \tag{19}$$

Equation (19) will be the mathematical model of the two phase diffusion through a porous medium. It is a nonlinear differential equation since the porosity and permeability depends on pressure. That is, $\emptyset = \emptyset(p)$ and $k = k(p)$. Equation (19) is too complex to solve analytically, therefore, it will be solved using the Finite Difference Method (FDM).

2.2. Model Discretization

Discretization using the backward forward difference scheme [14, 15] : The backward-forward scheme is an explicit finite difference scheme, giving a first order convergence in time and second order convergence in space [16]. The spatial interval [0, R] and time interval [0, T] are partitioned into respective finite grid as follows:

$$\mathrm{r_i}=(\mathrm{i}-1)\,\Delta\mathrm{r}, \qquad \mathrm{i}=1,2\ldots\mathrm{I}+1, \ \text{where}\ \Delta\mathrm{r}=\frac{\mathrm{R}}{I}. \tag{20}$$

$$\mathrm{t_n}=(\mathrm{n}-1)\,\Delta\mathrm{t}, \qquad \mathrm{i}=1,2\ldots\mathrm{N}+1, \ \text{where}\ \Delta\mathrm{t}=\frac{\mathrm{T}}{\mathrm{N}}. \tag{21}$$

The numerical solution is an approximation to the exact solution that is obtained using a discrete representation to the Partial Differential Equation (PDE) at the grid point r_i on the discrete spatial mesh at every time level t_n.The numerical solution at a grid point $p(r_i, t_n)$ is denoted by the symbol p_i^n and the derivatives are replaced by suitable difference quotients(to get an algebraic equation which can be solved). The backward-forward analog is;

Forward difference

$$\Delta p_i^n=p_{i+1}^n-p_i^n. \tag{22}$$

Backward difference

$$\nabla p_i^n=p_i^n-p_{i-1}^n. \tag{23}$$

Central difference

$$\delta^2 p_i^n=p_{i+1}^n-2p_i^n+p_{i-1}^n=\Delta\left(\nabla p_i^n\right). \tag{24}$$

$$\Delta(\nabla)=\nabla(\Delta).$$

Thus, substituting equation (22) - (24) into equation (19) results to

$$\frac{p_i^{n+1}-p_i^n}{\Delta t}=\frac{\alpha k\left(p_i^n\right)}{\emptyset\left(p_i^n\right)}\left[\frac{1}{k\left(p_i^n\right)}\frac{\partial k\left(p_i^n\right)}{\partial p}\frac{\Delta p_i^n}{\Delta r}\frac{\nabla p_i^n}{\Delta r}+\frac{\Delta p_i^n+\nabla p_i^n}{2\Delta r}+\frac{\delta^2 p}{(\Delta r)^2}\right]. \tag{25}$$

Along the left boundary

$$\Delta p_1^n=\frac{p_1^n-p_{1-1}^n}{\Delta t}=0. \tag{26}$$

And along the right boundary

$$\nabla p_I^n=\frac{p_{I+1}^n-P_I^n}{\Delta r}=0. \tag{27}$$

These conditions together with (25) results to

$$p_i^{n+1}=p_i^n+\frac{\alpha\Delta t}{(\Delta r)^2}\frac{k(p_i^n)}{\emptyset(p_i^n)}\left[\frac{1}{k\left(p_i^n\right)}\frac{\partial k\left(p_i^n\right)}{\partial p}\left(\Delta p_i^n\right)\left(\nabla p_i^n\right)+\frac{\Delta r}{2r}\left(\Delta p_i^n+\nabla p_i^n\right)+\delta^2 p\right]. \tag{28}$$

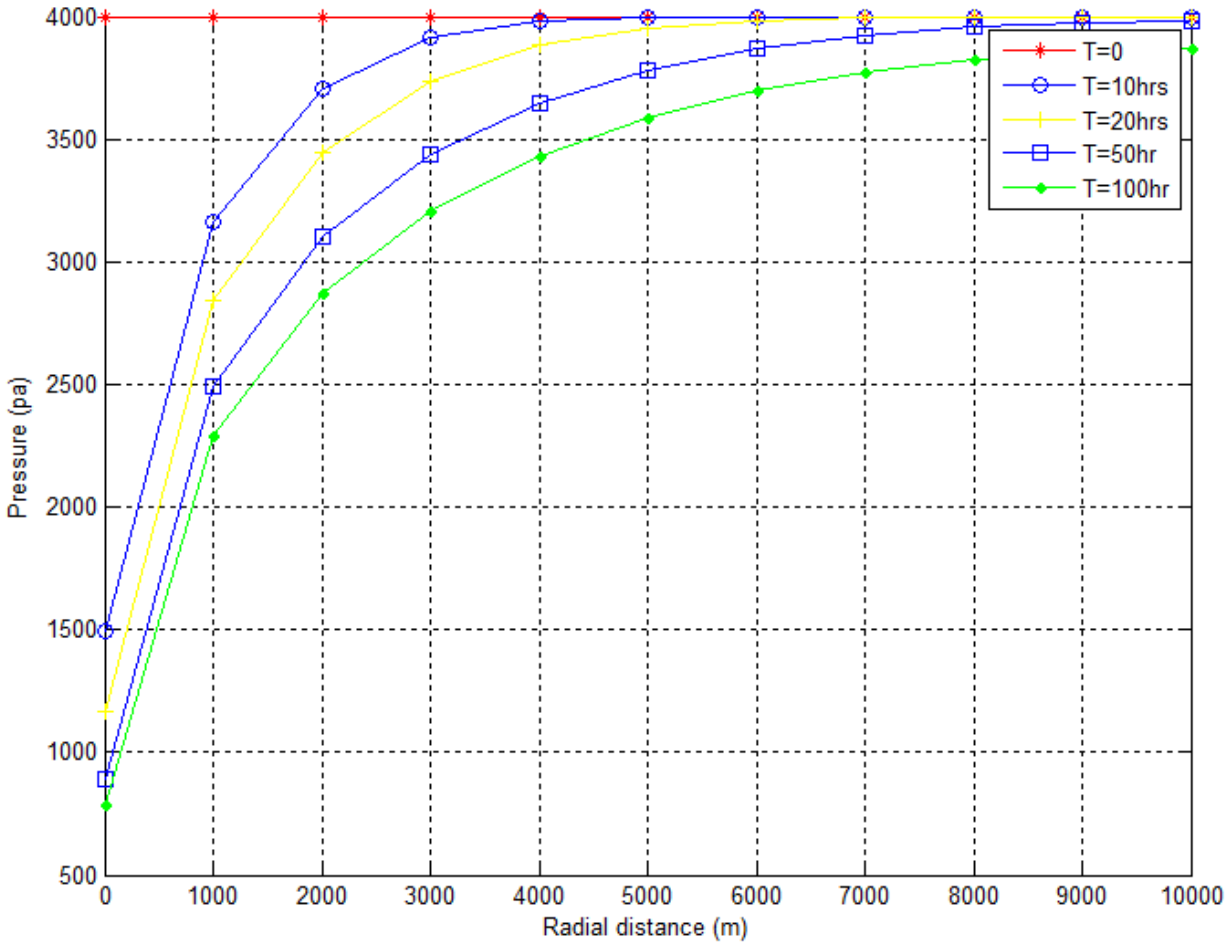

Figure 3: Pressure Distribution for the Linear Porosity and Permeability Model.

$p_i^1 = f(r)$, for this simulation,

$$p_i^1 = 4000\cos(\pi r_i). \tag{29}$$

For $i = 1$,

$$p_1^{n+1} = p_1^n - H\frac{k(p_1^n)}{\emptyset(p_1^n)}\left(\left(p_2^n - p_1^n\right) - \frac{p_2^n - p_1^n}{2}\right). \tag{30}$$

For $i = 2$ to I,

$$p_i^{n+1} = p_i^n + H\frac{k(p_i^n}{\emptyset(p_i^n)}\left[\frac{1}{k\left(p_i^n\right)}\frac{\partial k\left(p_i^n\right)}{\partial p}\left(\Delta p_i^n\right)\left(\nabla p_i^n\right) + \frac{1}{2(i)}\left(\Delta p_i^n + \nabla p_i^n\right) + \delta^2 p\right]. \tag{31}$$

For $i = l + 1$,

$$p_{I+1}^{n+1} = p_{I+1}^n + H\frac{k\left(p_{I+1}^n\right)}{\emptyset\left(p_{I+1}^n\right)}\left[\frac{1}{2\,(I+1)} - 1\right]\left(p_{I+1}^n - p_I^n\right), \tag{32}$$

where,

$$H = \frac{\alpha\Delta t}{(\Delta r)^2} \text{ and } \frac{\Delta r}{r} = \frac{1}{i}. \tag{33}$$

The numerical solution is obtained by calculating p_i^{n+1} recursively for n = 1,. . .,N by the use of equation (29), (30), (31) and (32). In the usual diffusion equation, where k and $\emptyset$ are constants, equation (30), (31), and (32) are completely ready for simulation. In this

Table 1: Values for Rock and Fluid Properties.

Parameter	Value	Unit
Viscosity, μ	0.001	Pas
Reference porosity, $\emptyset_{re}$	0.5	_
Reference permeability, k_{re}	$1.0e^{-10}$	m^3
Reference pressure, ρ_{re}	4000	Pa
Total compressibility, c	$5.07e^{-14}$	Pa^{-1}
Relative permeability, k_r	0.1	_
Oil saturation, s	0.5	_
Length of reservoir, R	10000	M

work, these functions are functional of pressure. To make any further head way one must make models of the functional dependence on pressure. This is the major departure from the known diffusion equation. Another departure from the known diffusion equation is the presence of the square on the derivative of pressure with respect to time.

2.3. The linear porosity, $\emptyset$ and permeability, k, model

The linear porosity, $\emptyset$ and permeability, k, model is giving as:

$$\emptyset = \emptyset_{re}\frac{p}{p_{re}}, k = \frac{k_{re}p}{p_{re}}. \tag{34}$$

2.4. Simulation Parameters

The simulation parameters and the conversion factor to SI units are given in Table 1 [17].

2.5. Simulation Procedure

The simulation equations (equation (25)-(34)) are coded in a MATLAB programing language. The linear porosity, $\emptyset$ and permeability, k model is contained in separate scripts as phi and kappa respectively and are combined to give the required model. The input parameters contained in Table 1 were initialized with their corresponding values in the program. The simulation procedure is as follows;

- start program: discretize the diffusion type (equation (25) - (28))
- input the parameters needed to described the reservoir (Table 1)
- set the initial conditions (equation (29))
- set the boundary conditions (equation (30) and (32))
- calculate porosity and permeability from previous step (equation (34))
- calculate the pressure at each time step from the result obtained in the previous step using equation (31)
- display results
- plot graphs
- end program.

3. Results and Discussions

When the porosity, $\emptyset$ and permeability, k are constant the simulation equation (19) reduces to a linear regular diffusion type equation (7). The concentration of the diffusing agent should have a Gaussian distribution while the Gaussian picks keep reducing in time [18]. Our simulation equation only appears like a diffusion type equation since pressure is not a diffusing agent rather an agent that causes flow of fluids. If a fluid is to flow towards the origin (wellbore) the pressure distribution should follow the Gaussian curves for $r < 0$. That is, the pressure distribution curves should be a mirror reflection of concentration diffusion curves. Our simulation results reproduce this situation very well as shown in Figure 1. For $t_i > t_j$, we expect the subsequent pressure distribution curves to lie below those of t_j. This is correctly obtained in the simulation (Figure 1).

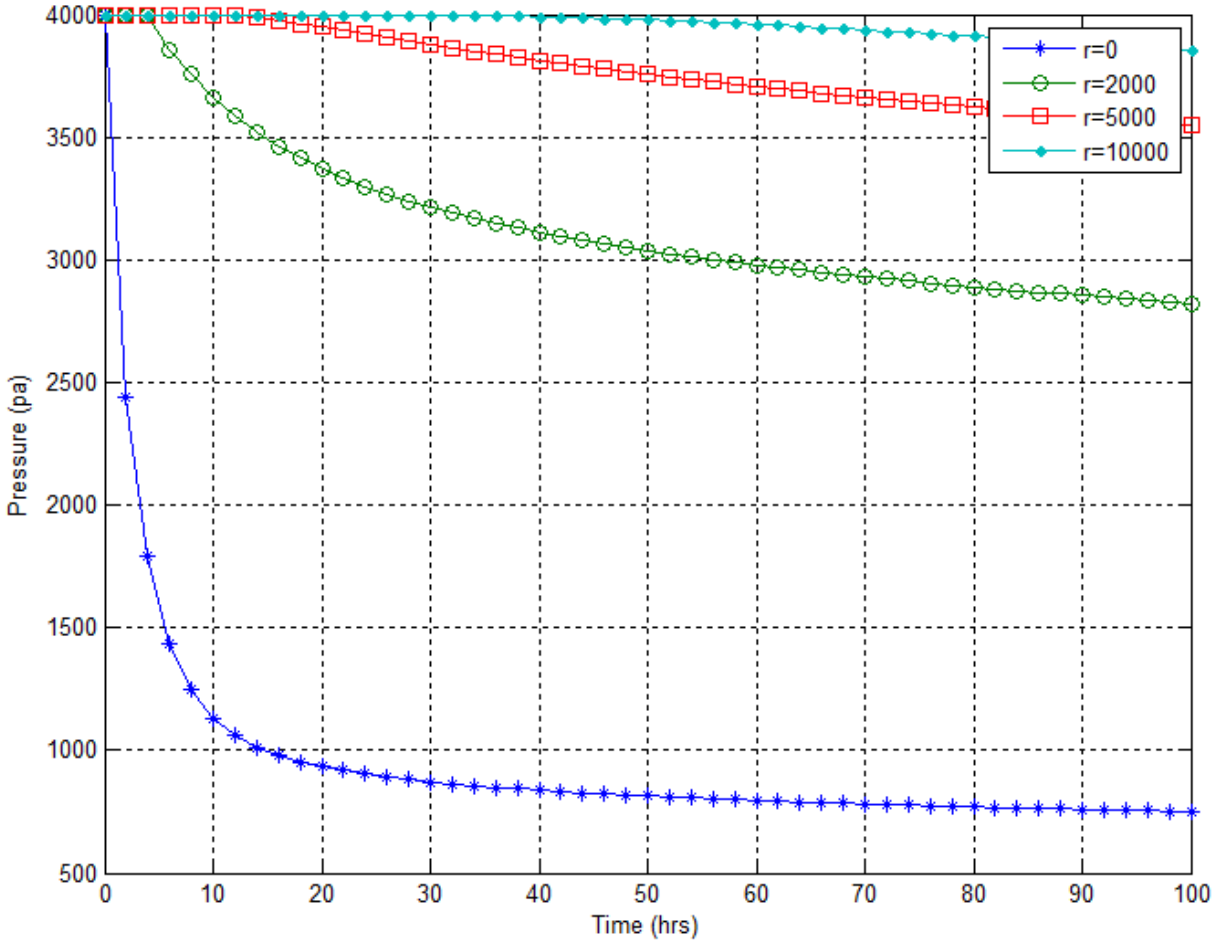

Figure 4: Pressure Profile for the Linear Porosity and Permeability Model.

Similarly, for fixed locations, we expect the pressure profile to follow a Gaussian decrement with time as shown in Figure 2. The constant porosity and permeability model is a baseline model with known analytic result. The correct reproduction of the simulation results gives us the confidence that while the porosity and permeability are no more constants, the simulation results may contain useful physics.

The linear porosity and permeability model has both porosity and permeability increasing with increasing pressure and vice versa. Both the porosity and the permeability will be increasing with increasing distances from the wellbore and decreasing with increasing simulation time in the same way the pressure will. A reservoir with such a model will have its porosity and permeability decreasing as fluid flows towards the well and with time fluid flow towards the well will cease. Such a situation can occur in a reservoir when there is blockage of pores by debris from the fluid flows. The curves from the plots in Figures 3 and 4 shows similar trend of pressure decline to that of Figure 1 and Figure 2 but the difference is quite clear when you compare the pressure drops at each location. At each location and time, the pressure distribution curve form Figure 3 recorded a higher value than that from Figure 1.

Also, from the pressure profiles it ca be observed that the pressure drop with time was faster in Figure 2 than in Figure 4. From Figure 2, at the end of the simulation time pressure at location r=0 dropped to 500 Pascal, r=200 meters dropped to below 2500, r=500 dropped to below 3500 but in Figure 4, r=0 was still above 500 Pascal, r=200 above 2500 and r=500 above 3500. Thus with a linear porosity and permeability model a reservoir will experience lesser pressure drop than that with a constant porosity and permeability model.

4. Conclusion

A nonlinear pressure diffusion–type equation with pressure dependent porosity and permeability model was derived and simulated using an explicit backward-forward finite difference scheme providing numerical solutions with the aid of MATLAB. The numerical results predicted the pressure distribution in a petroleum reservoir under the constant porosity and permeability model and also under a linear porosity and permeability model. The results showed that the pressure transient was slower with the linear porosity and permeability model than with the constant porosity and permeability model. This means that pressure dependency of porosity and permeability does affect the pressure gradient, profile and fluid yield at the wellbore. This work can be applied in the oil and gas industry to predict pressure behavior in reservoirs and make investment decisions, production and maintenance decision.

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References

[1] Y. Jin & K. Chen, "Fundamental equations for primary fluid recovery from porous media", Journal of fluid Mechanics **860** (2019) 300. https://doi.org/10.1017/jfm.2018.874
[2] L. Yan, D. P. Jonathan, A. W. David & E. Bruce, "Nonlinear pressure diffusion in a porous medium. Approximate solutions with applications to permeability measurements using transient pulse decay method", Journal of Geophysical Research: Solid Earth **106** (2001) B1. https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2000JB900344
[3] V. K. Evgenii, S. T. Mikhail, P. R. Evgenii & V. P. Vladimir, "Effect of effective pressure on the permeability of rocks based on well testing results", Energies **14** (2021) 8. https://doi.org/10.3390/en14082306
[4] E. Selcuk, S. J. Fowler, H. Virginie, & L. Ben, "An analytical model of porosity-permeability for porous and fractured media", Transport in Porous Media **120** (2017) 327. https://doi.org/10.1007/s11242-017-0923-z
[5] L. Chunguang, S. Guanhua, G. Mingwei & W. Shuilin, "Permeability of porous media under confining pressure", Geophysics Journal International **216** (2019) 3. https://doi.org/10.1093/gji/ggy531
[6] A. E. Scheidegger, *The physics of flow through porous media (3rd Edition)*, University of Toronto Press, Toronto, (2020), pp. 1-53. https://www.degruyter.com/document/doi/10.3138/9781487583750/html
[7] L. Shen, T. Cui, H. Liu, Z. Zhu, H. Zhong, Z. Chen, B. Yang, R. He & H. Liu, "Numerical simulation of two-phase flow in naturally fractured reservoirs using dual porosity method on parallel computers", Proceedings of the International Conference on High Performance Computing in Asia Pacific Region, Guangzhou, China, (2019), pp. 91-100. https://doi.org/10.1145/3293320.3293322
[8] S. Gasmi, & F.Z. Nouri, "Numerical simulation for two phase flow in a porous medium", Boundary Value Problem **2015** (2015) 7. https://doi.org/10.1186/s13661-014-0256-6 Numerical Treatment of Two-phase Flow in Porous Media Including Specific Interfacial Area
[9] M. El-Amin, J. Kou, A. Salama & S. Sun, "Numerical treatment of two-phase flow in porous media including specific interfacial area", Science Direct **51** (2015) 53. https://doi.org/10.1016/j.procs.2015.05.306
[10] M. C. Jose, P. Stefano, J. Santos, N. Ayman, N. A. Qadrouh & H. Almalki, "Numerical simulation of two phase fluid flow", Journal of Petroleum Exploration and Production Technology **4** (2014) 233. https://doi.org/10.1007/s13202-014-0109-y
[11] C. G. Nmebu, "Modelling the pressure distribution in a reservoir undergoing MEOR for a 2-dimensional flow system", International journal of Emerging Technology and Advanced Engineering **4** (2014) 6. https://www.researchgate.net/publication/263315861_Modeling_the_Pressure_Distribution_in_a_Reservoir_Undergoing_MEOR_for_a_2-Dimensional_Flow_System
[12] A. Opoku, K. D. Ohenewaa & T. B. Eric, "A Comparative study between finite difference explicit and implicit method for predicting pressure distribution in a petroleum reservoir", International Journal of Engineering science and Technology **11** (2019) 23. https://doi.org/10.4314/ijest.v11i4.3
[13] Z. Chen, G. Huan & Y. Ma, "Computational methods for multiphase flows in porous media", Computational Science and Engineering Series, Philadelphia, PA, USA, (2006), pp. 1-282. https://doi.org/10.1137/1.9780898718942
[14] M. K. Jain, S. R. K. Iyengar & R. K. Jain, "Computational methods for partial differential equations", 2nd Edition, Newyork, New Age International Publishers (2016) 240. https://www.amazon.com/Computational-Methods-for-Partial-Differential-Equations/dp/812243973X
[15] L. B. Winrich, *An Explicit method for numerical solution of a nonlinear diffusion equation*, Ph.D. dissertation, Mathematics Department, Iowa State University of Science and Technology, Ames, Iowa, 1968. https://dr.lib.iastate.edu/entities/publication/a132deb4-9568-4883-aa63-e828f9eef9b0
[16] S. Schmelter & M. Bar, "Numerical simulation validation and analysis of two phase slug flow in large horizontal pipes", Journal of Flow Measurement and Instrumentation **73** (2020) 101722. https://www.sciencedirect.com/science/article/pii/S0955598620300704
[17] T. Ahmed & P. D. McKinney, *Advanced reservoir engineering (1st Edition)*, Oxford, Gulf Professional Publishing, Elsevier, Burlington, United states of America, (2005), pp. 1-363. https://doi.org/10.1016/B978-0-7506-7733-2.X5000-X
[18] M. A. Abdullah, A. Panda, S. Gupta, S. Joshi, A. Singh, & N. Rao, "Multi-scale method for modeling and simulation of two phase flow in reservoir using MRST", Petroleum and Coal **61** (2019) 3. https://research.vit.ac.in/publication/multi-scale-method-for-modeling-and-simulation-of-two-phase-flow-1

APPENDIX A.

Simulation Code

Script1
function phi = wfunc(p)
phi = 0.18*(p)/4000;

```
end
script2
function kappa = wfunc2(p)
kappa=1e-10*p/4000;
end
%Numerical simulation of pressure diffusion type equation in a two phase
%porous media with Linear porosity and permeability model
%parameters needed to solve the equation
J=1e-10; %reference permeability
G=0.18; %reference porosity
U=10e-3; %viscosity in pa.s
Kr=0.1; % relative permeability
so=0.5; %oil saturation parameter
c=5.7e-14; %isothermal compressibility 1/pa
R=10000; %total radial distance of reservoir in meters
dr=1000; %space step
I=R/dr; %space step interval
T=100; %total time in hrs
dt=2; %time step interval
N=T/dt; %time step
H=(Kr*dt)/(c*so*U*dr*dr);
%p=zeros(I+1,N+1);
%initial condition
for i=1:I+1;
r(i)=(i-1)*dr; %space discretisation
p(i,1)=4000*cos(pi*r(i));
end
for n=1:N;
t(n)=(n-1)*dt; %time discretisation
p(1,n+1)=p(1,n)+(H*wfunc2(p(1,n))/wfunc(p(1,n)))*(p(2,n)-p(1,n)-(p(2,n)+p(1,n))/2); % well bore pressure
for i=2:I
A(i,n)=(p(i+1,n)-p(i,n)+p(i,n)-p(i-1,n))/2;
B(i,n)=((p(i+1,n)-p(i,n))*(p(i,n)-p(i-1,n)));
C(i,n)=p(i+1,n)-2*p(i,n)+p(i-1,n);
p(i,n+1)=p(i,n)+(H*wfunc2(p(i,n))/wfunc(p(i,n)))*(C(i,n)+A(i,n)/i+(J/(4000*wfunc6(p(i,n))))*B(i,n));
end
p(I+1,n+1)=p(I+1,n)+H*(wfunc2(p(I+1,n))/wfunc(p(I+1,n)))*(-p(I+1,n)+p(I,n)+(p(I+1,n)-p(I,n))/(2*(I+1)));
end
disp(p)
figure(3)
%r=(i-1)*dr;
plot(r,p(:,1),'*-r',r,p(:,6),'o-b',r,p(:,11),'+-y',r,p(:,round((N+1)/2)),'s-k',r,p(:,N+1),'.-g')
%title('Pressure distribution in a reservoir at different time')
legend('T=0','T=10hrs','T=20hrs','T=50hr','T=100hr')
xlabel('Radial distance')
ylabel('Pressure (pa)')
grid on
figure(4)
t=0:dt:T;
plot(t,p(1,:),'*-',t,p(round(I/3),:),'o-',t,p(round((I+1)/2),:),'s-',t,p(I+1,:),'.-')
%title('Pressure distribution in a reservoir at various radial distances for 100hrs from the well')
legend('r=0','r=2000','r=5000','r=10000')
xlabel('Time (hrs)')
ylabel('Pressure (pa)')
grid on
%program ends
```